Flushes on Board

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Bib Ladder, shaking his head in wonder, left the poker table on a course straight to my booth. I know him well enough to realize silence was the operative word, so to speak, for his condition at the time. He'd talk when he was ready. Eventually, he asked, "*Professor, what are the chances of making a flush when a player is holding two suited cards*?" He didn't need to say anything else. We've all watched big hands go down in flames to someone holding two small suited cards. Here is a summary of what I told Bib.

First we look at the chances for a board to allow a flush without any information about anyone's hand. There are $C(52,5) = 2,598,960$ boards altogether. There are 5,148 boards with all five cards in the same suit. This follows because there are four choices of suit and $C(13,5) = 1,287$ ways of choosing five cards from 13.

If there are four suited cards on board, there are four choices for the suit, $C(13,4) = 715$ choices for four cards of the suit, and 39 choices for the remaining card. Multiplying gives 111,540 boards with four suited cards.

If there are three suited cards, there are four choices for the suit allowing the flush, $C(13,3) = 286$ choices of three cards from that suit, and $C(39,2) = 741$ choices for the other two cards from the remaining 39 cards. Multiplying yields 847,704 boards with three suited cards.

Summing the above numbers produces 964,392 boards that allow a flush. Dividing by the total number of possible boards gives us a probability of .371 that the board allows a flush. This is slightly less than 3/8, so that you should see boards allowing a flush about three out of every eight hands.

The above figures apply to someone who is watching the board and has no knowledge of any of the players' hands. Off equal or more interest, however, is the situation for one of the players in the game. Suppose a given player has two suited cards. What is the probability that this particular player will make a flush? We lose no information by assuming the player has two hearts in her hand.

From the player's viewpoint, there are $C(50,5) = 2,118,760$ possible boards since the board is formed by choosing five cards from 50 unseen cards. The number of hearts unaccounted for is 11. One possibility is that five hearts form the board. This can happen in $C(11,5) = 462$ ways.

If four hearts end up on board, there are $C(11,4) = 330$ ways of choosing four hearts from 11 and 39 ways to choose the remaining card. Multiplying the two numbers gives 12,870 boards with four hearts.

Finally, there are $C(11,3) = 165$ ways to choose three hearts from 11, and there are $C(39,2) = 741$ ways to choose two cards from the remaining 39. Multiplying gives us 122,265 boards with three hearts.

Adding the three numbers produces 135,597 boards giving the player holding two hearts a flush. The probability is then .064 that she makes a flush. This is approximately 1 in 15.6. *In terms of odds, the odds against a player making a flush when holding two suited cards are about 14.6-to-1.*

After completing the calculations, Bib observed, "Those numbers make it pretty hard to see how someone can play 3-7 of spades against three other players when the betting is capped before the flop."

"Is that what happened, Bib?"

"Yep, and I even flopped a K with my pocket Ks. I had a bad feeling when the turn and river came spades."

"Bib, let's take a look at the corresponding numbers for Omaha."

"If you insist, Professor."

In terms of an outside observor watching boards in Omaha, everything is the same. That is, the probability that the board allows a flush still is .371. The situation from a player's standpoint, though, is going to be different because of the additional cards in a player's hand. If an Omaha player has two hearts in her hand and two other cards that are not suited, our intuition tells us that her chances of making a flush should be higher than in hold'em because two non-hearts have been removed from the deck. In addition, the total number of possible boards is $C(48, 5) = 1,712,304$ which is a smaller number than for hold'em..

Indeed, this turns out to be the case as the probability is .072, or about 1 in 14, as compared to .064. That is about a 12% improvement in the chances of making a flush for Omaha in a similar situation.

If we now give an Omaha player three suited cards in her hand, it is intuitively clear that her chances of making a flush are going to diminish. It turns out to be the case that the probability of having a board that gives her a flush drops to .054 or about 1 in 18.5.

The worst possible situation for a player, with regard to flushes, is to have four cards all in the same suit. Under these conditions, the probability of having a board that gives her a flush is .039 or about 1 in 25.4.

Thus, we have seen that if an Omaha player has two, three or four suited cards, then her probabilities of making a flush are .072, .054 or .039, respectively. However, there is one important scenario we have omitted for Omaha. What happens when an Omaha player has two suits with two cards of each suit in her hand? For example, does a pair of double suited aces have a lot of extra value just from the flush point of view?

As a matter of fact, we can use our earlier work for this situation to get an answer quickly. The reason for this is because there is no overlap between boards allowing a flush in one of the suits and those allowing a flush in the other suit. Thus, we can simply double the number of boards allowing a flush for a hand with two hearts. It then follows that the probability that an Omaha player holding double suited flush possibilities achieves a flush is .143 or about 1 in 7. This is a reasonably significant number.