Realizing Quads

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Anyone who has dealt with me in person or via e-mail knows that I am a stickler for precision. Mathematicians attempt to provide precise answers whenever they can. The first requisite step in the process is a precisely phrased question. People who turn to a mathematician for an answer to a problem frequently provide a description of the problem that is open to many interpretations. I have done enough consulting to confidently say that the biggest hurdle for most consulting is understanding what the problem really is. This brings me to this month's question.

A player holding 7-7 called a raise and then saw a flop of A-K-9. He folded to aggressive betting and watched with dismay as the remaining sevens came on the turn and river. He had missed quads because even though the board ultimately contained two sevens, they came too late for him. This prompted him to ask me what the chances are of making quads when one holds a pocket pair and the board contains the other two cards of that rank.

Given my introductory paragraph, you probably realize that his question is not precise. It may sound precise, but if you begin to think about it carefully, you will realize that any answer you might give is going to have 'ifs' and 'buts' in it. His question is closely related to a homework question I always use in the mathematics of gambling course I sometimes teach. The homework question is the following: Given a fixed board in Omaha or hold'em, in how many ways can the board be dealt?

This question is interesting for several reasons. First, when calculating the probabilities for certain scenarios to play out in hold'em or Omaha, the order in which the cards forming the board appear clearly plays a role (witness the player above missing quads). Second, the question has two reasonable interpretations so that students have to think through why one interpretation makes sense and the other interpretation does not. Third, many counting questions involve either the concept of an ordered or an unordered problem. This question requires a mixture of both concepts.

Let's return to the original question about what are the chances of realizing quads given that the board allows them. That is, suppose a player holds x-x and the board contains x-x, then what are the chances the player will realize his quad x hand? The only precise question emerging is the following: Given that a board contains x-x, what is the probability that they come on the turn and river? Now we see why we need the answer to the homework problem stated above. Let's answer that question first.

Many students give an answer that essentially goes as follows. There are five cards comprising the board. So the firsy card may be any of five cards, the next card may be any of four, the next card any of three, the turn card any of two, and the remaining card is determined. Multiplying gives us 120. Hence, the board can appear in 120 ways. This is a valid line of reasoning and technically correct, but it is a silly interpretation! Almost every question dealing with boards does not care in which order the three cards comprising the flop occur. Instead, what is of interest is the three cards making up the flop. We obtain the cards in the flop by choosing three cards from five, that is, the flop can be chosen in $C(5,3) = 10$ ways. There are then two choices for the turn card and the river card is determined. According to this interpretation, the board may be dealt in 20 ways.

The latter interpretation is the sensible one to use for most problems including the problem we are discussing. There are then two ways to deal the board so that both cards of rank x appear as the turn and river card. This means the probability is 2/20, which is equal to 1/10, that when the board allows quads neither card of rank x appears on the flop. To complete the possibilities, there is a probability of 3/10 that the player flops quads, and a probability of 6/10 that the player flops trips.

It is clearly incorrect to then say that the chances of the player realizing his quads is 9/10 because that is the probability he flops at least one card of rank x. To try to claim that, we would be making three dangerous assumptions:

1. The player will not fold his pair of rank x preflop no matter how the betting progresses;
2. The player will go all the way to the river if he flops a set; and
3. The player will fold his hand if he does not flop a set.

It is easy to see that the preceding assumptions do not take into account the rank x of the player's pair. If a player has Aces or Kings, he almost certainly is going to see the flop if there is a flop. His opponents may fold to a raise so that he wins outright without a flop. If he flops a set, there is a reasonable chance he may win the pot right there. Again the player will never know that he would have made quads. If the player does not flop a set, there is a good chance that he will see the turn with Aces or Kings. Thus, when he makes the set on the turn he almost surely will see the river; if his opponents don't surrender after the turn, and his quads will see the light. To try to assign meaningful probabilities to the occurrences just discussed leads us into mushy thinking and a lot of guesswork.

With nines through queens, I suspect there is a better chance the player would realize his quads because it is more likely the betting is going to lead to a flop. He now has a 90% chance of flopping at least a set and this usually keeps him in until the end unless the other players fold. With sixes through eights, the situation is similar to nines through queens except there is an increased chance the player may fold his pair to certain betting situations. With small pairs, twos through fives, I think it is more likely a player will fold under a lot of betting pressure. So the chances of a player realizing quads in this situation is diminished.

In conclusion, the one precise fact we can state is that when a player is holding x-x and the board is going to produce x-x for quads, there is a 90% chance the player flops a set or better if he stays for the flop. Trying to be more precise than this and take into account the rank x leads to speculative results. One could say something reasonable by taking into account the characteristics of the game in hand, but would vary depending on many variables.