# Are these Astronomical Odds?

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In September I received an e-mail message: “I wanted to know if you could tell me what the chances are of what happened to me about 20 minutes ago on an online poker site. The flop came 10♠, J♠, K♠. I held two tens and my opponent held the A♠, Q♠. The turn card was the 3♡ and the river card was the fourth 10. In other words, I had quad tens and lost to a royal flush. Isn’t this almost a statistical impossibility? I mean the chances of this have to be astronomical.”

This article is devoted to variations on his question. The first problem with his question is that it is ill-posed as it stands. What I mean is that it has several interpretations. This is a common occurrence when people from the general public ask probability questions. They are not familiar with the standard way to frame questions so that there is only one interpretation. Let’s take a look at some of the possible interpretations.

One precise interpretation is the following. Suppose one player has been dealt 10-10 and another player has been dealt suited A-Q of a suit different from either 10. What is the probability that one player ends up with quad tens and the other player end up with a royal flush? The total possible number of boards is $C(48, 5) = 1,712,304$. Boards that produce quad tens and a royal flush have four of the cards completely determined. The remaining card can be any of 44 cards. Thus, the probability is 44 divided by 1,712,304 which is 1/38,916. Thus, the chances of it happening are small but not astronomically small.

Let’s change the scenario somewhat. Suppose a player has looked at his hand and finds 10-10. Another precise question is what is the probability that the player holding 10-10 ends up with quads and loses to a royal flush with three royal cards on board? First we calculate the total number of completions of the player’s hand to a 10-handed semideal. There are $C(50, 5) = 2,118,760$ ways to choose the board cards, there are $C(45, 18) = 1,715,884,494,940$ ways to choose 18 cards for the other nine hands, and there are $17!! = 34,459,425$ ways to divide the 18 chosen cards into nine hands. Multiplying all of these numbers yields 125,278,874,084,144,416,856,220,000 semideal completions.

All we need to do now is to count the number of semideal completions with two tens on board and a royal flush, where the player with the royal flush has two cards in her hand. The player with the royal flush must use one of the tens on board. There are two choices for the suit of the royal flush and there are six choices for the other two royal cards in the suit. This gives us 12 choices so far. The fifth board card can be any of 44 choices. This gives 528 possible boards. An additonal hand is now fixed in order for a player to have a royal flush. This means we are choosing eight hands from 43 cards. We then have

$$528C(43, 16)15!! = 283,816,286,697,829,485,600.$$

We divide by the total number of semideal completions and get a probability of .00000226 which is about 1 in 441,408.

In the first scenario above, we are asking what the chances of something happening are when two hands are specified ahead of time. In the second scenario, we are specifying only one hand ahead of time and the reader will note that the probability has decreased by a factor of about 11.

If we do not specify any hand and ask what are the chances on the next hand that player A is dealt 10-10, makes quad tens, and loses to a royal flush, where the winning player has two of the royal cards in her hand, we would guess that the probability is even smaller. Going through arguments similar to those given above, we find that the probability of this happening is .0000000103 or about 1/97,551,242. Now this is getting astronomical!

It is extremely unlikely that the preceding scenario will happen to us on the next hand. However, we could ask what the chances are that it will happen to us over an extended playing session. If we play a session of 100 hands, the probability that it will happen to us is about .00000103 making it still highly unlikely. If we play a session of 200 hands, the chances of it happening are only about 1 in 500,000. If we play a session of 1,000 hands, the probability of it happening to us is about 1 in 100,000. You can see tit is quite unlikely to happen to an individual player, but we know that highly unlikely events take place frequently. There always is a small chance you will be involved in a memorable highly unlikely event sometime during your next poker session.