Opponents Holding Bigger Pairs

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Bib Ladder has been after me for a long time to update my earlier information about the likelihood of a player holding a pocket pair in facing an opponent with a bigger pair. He wants me to expand the information from nine opponents to anywhere from one through nine opponents. I have resisted starting the task because I knew it would be time consuming. However, I have succumbed to his constant, though gentle, enquiries regarding my progress. This article contains the information in four separate tables. It is valuable information and represents about fifty hours of work.

Opponents	Three			Two		One
Bigger Ranks	3	2	1	2	1	1
KK	-	-	.01467	-	.00980	.00490
QQ	-	.00016	.02902	.00005	.01954	.00980
JJ	.00000	.00047	.04307	.00016	.02923	.01469
TT	.00000	.00093	.05681	.00031	.03887	.01959
99	.00001	.00153	.07024	.00052	.04846	.02449
88	.00002	.00229	.08338	.00078	.05799	.02939
77	.00003	.00318	.09617	.00109	.06748	.03429
66	.00005	.00422	.10875	.00146	.07691	.03918
55	.00008	.00539	.12100	.00188	.08629	.04408
44	.00011	.00670	.13296	.00234	.09561	.04898
33	.00015	.00814	.14462	.00287	.10489	.05388
22	.00020	.00971	.15600	.00344	.11411	.05878

ONE THROUGH THREE OPPONENTS

The first table is for one, two, and three opponents; the remaining tables are set up in exactly the same way. The leftmost column gives the rank of the pair that our given player is holding. The row labelled "bigger ranks" contains numbers corresponding to the number of bigger ranks for which opponents hold at least one pair of that rank. A few examples should make the situation perfectly clear.

Suppose our given player is facing three opponents and finds 8-8 when she peeks at her cards. Find the row corresponding to 8-8. There are three columns for three opponents because she could be facing as many as three different pairs of bigger ranks than 8. However, you will note that the probability of this is .00002, which is about 1 in 50,000. So it is extremely rare. The probability that she is facing pairs of two bigger ranks is .00229, which is about 1 in 437. The probability that she is facing one bigger pair is .08338, which is about 1 in 12 times.

Suppose our given player is playing heads-up and is dealt a pair of jacks. Because she is facing only one player, there is no possibility of more than one

larger pair against her. Thus, there is only one column of entries for one opponent. In this case, the probability is .01469 that her opponent has a larger pair. This is about 1 in 70 so that normally a player in this situation is ahead before the flop.

Opponents	Five			Four		
Bigger Ranks	3	2	1	3	2	1
KK	-	-	.02445	-	-	.01957
QQ	-	.00052	.04785	-	.00031	.03851
JJ	.00001	.00153	.07025	.00000	.00093	.05684
TT	.00004	.00301	.09166	.00001	.00183	.07456
99	.00009	.00493	.11211	.00004	.00301	.09170
88	.00018	.00726	.13163	.00007	.00447	.10825
77	.00030	.00999	.15024	.00012	.00618	.12423
66	.00048	.01308	.16796	.00020	.00814	.13965
55	.00072	.01651	.18483	.00030	.01034	.15453
44	.00101	.02027	.20086	.00042	.01277	.16886
33	.00138	.02433	.21608	.00057	.01542	.18266
22	.00182	.02866	.23051	.00076	.01827	.19594

Four and Five Opponents

I would be remiss if I did not provide more information about the entries. First, let me emphasize that the column headings do not correspond to the number of bigger pairs facing you. They tell us the number of bigger ranks for which there are pairs dealt. Thus, if you hold 9-9 and are facing five opponents, one opponent holding J-J, another opponent holding K-K, and no other opponent pairs, then this contributes to the entry .00493 in the table for four and five opponents. However, one opponent holding J-J, another holding Q-Q, another holding the other Q-Q, and no other opponent pairs also contributes to the same entry .00493. So we only know the number of bigger ranks with pairs dealt and not the pattern of how many pairs of each rank have been dealt.

Another important feature is that the columns headed "1" and "2" mean exactly one and two, respectively, larger ranks with pairs dealt; whereas, the column headed "3" means three or more larger ranks have been dealt with pairs of those ranks. However, in the latter case, the probabilities are so small that it is not important whether you are facing pairs with three larger ranks or even more pairs with larger ranks.

Opponents	Seven			Six		
Bigger Ranks	3	2	1	3	2	1
KK	-	-	.03419	-	-	.02932
QQ	-	.00109	.06621	-	.00078	.05709
JJ	.00003	.00317	.09614	.00002	.00228	.08335
TT	.00012	.00616	.12409	.00007	.00446	.10816
99	.00030	.00997	.15012	.00018	.00725	.13158
88	.00059	.01451	.17434	.00035	.01063	.15364
77	.00102	.01973	.19682	.00060	.01453	.17440
66	.00160	.02553	.21765	.00094	.01891	.19392
55	.00235	.03186	.23689	.00139	.02374	.21222
44	.00329	.03865	.25462	.00196	.02896	.22990
33	.00442	.04585	.27091	.00265	.03455	.24539
22	.00577	.05340	.28583	.00348	.04046	.26034

SIX AND SEVEN OPPONENTS

If you want the probability that you are facing one or more larger pairs, simply add the probabilities under the thee columns for your fixed pair. For example, suppose you hold 5-5 against six opponents. The sum of the three entries is .23735 so that the chance you are facing at least one bigger pair is about 1 in 4. Note that the entry for exactly one larger rank provides the bulk of the total. This is always the case because the probability of facing two or more larger ranks with pairs is small.

Opponents	Nine			Eight		
Bigger Ranks	3	2	1	3	2	1
KK	-	-	.0439	-	-	.03906
QQ	-	.00186	.08412	-	.00145	.07501
JJ	.00008	.00544	.12161	.00005	.00420	.10791
TT	.00028	.01044	.15519	.00020	.00811	.13772
99	.00071	.01662	.1857	.00048	.01304	.16442
88	.00137	.02397	.2132	.00094	.01888	.18799
77	.00230	.03218	.2380	.00160	.02550	.20838
66	.00354	.04114	.2603	.00250	.03280	.22555
55	.00510	.05071	.2801	.00366	.04068	.23947
44	.00700	.06076	.2977	.00510	.04905	.25008
33	.00925	.07118	.3133	.00685	.05782	.25733
22	.01186	.08186	.3269	.00891	.06690	.26115

EIGHT AND NINE OPPONENTS

Let me say a few words about the entries in general. The probabilities given have been determined using a method that produces the exact answer. In other words, they are not the result of some kind of approximation method. Of course, the exact values have been rounded off to five decimal places of accuracy.

This means, as a player, upon being dealt a pair, the tables contain the exact probabilities that you are facing pairs of bigger ranks.

Once the betting starts, you now are getting additional information and such information should not be ignored. If you have Q-Q against seven opponents, raise, and are reraised by someone you know would not reraise with anything other than K-K, A-A, or A-K suited, then this knowledge overrides the information from the table telling you that there is only a 1 in 16 chance you are facing K-K or A-A. You must base your action on all the available information as it develops throughout the hand.

Finally, let me say a few words about the derivation of the numbers in the tables. Let L denote the number of ranks bigger than the rank of the pair you are holding. For each column in the tables, there is a corresponding polynomial P in L. For each possible value of L, you substitute the value in P and divide by the total number of possible hands to get the appropriate probability. As an example, let's look at the entries for three opponents and the column headed by "2". Three opponents can be dealt hands in $C(50,2)C(48,2)C(46,2) = 1,430,163,000$ ways. The polynomial here is $-648L^3 + 113,616L^2 - 112,968L$. If you substitute the values of L from 1 through 12 and divide by the above number, you will obtain the numbers in the appropriate column.

Anyone interested in the detailed calculations will find them at my website one of these days. The current file entitled "Multiple Pocket Pairs" is now out of date and will be replaced as soon as possible with the details for the derivation of the above tables.