Opponents and Overcards: Part 2

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In this second, and last, part dealing with possible opponent kickers, we consider a player holding either K-x or Q-x, where the player's kicker has smaller rank than either K or Q, respectively. In the case of K-x, we shall consider from one through nine opponents, whereas, for Q-x we shall consider only from one through five opponents. The reason for the latter is that the chances of Q-x being dominated already are very high with just five opponents. The situation clearly worsens as the number of opponents increases

kicker	9	8	7	6	5	4	3	2	1
Q	.8479	.8033	.7494	.6850	.6089	.5474	.4157	.2955	.1576
J	.8637	.8216	.7696	.7069	.6315	.5646	.4360	.3119	.1673
10	.8783	.8390	.7888	.7278	.6534	.5818	.4560	.3281	.1771
9	.8920	.8557	.8070	.7478	.6744	.5990	.4756	.3442	.1869
8	.9046	.8716	.8242	.7669	.6948	.6160	.4949	.3601	.1967
7	.9163	.8868	.8405	.7851	.7144	.6330	.5139	.3759	.2065
6	.9270	.9013	.8558	.8024	.7333	.6500	.5325	.3916	.2163
5	.9368	.9151	.8702	.8189	.7514	.6669	.5508	.4071	.2261
4	.9458	.9284	.8837	.8346	.7689	.6837	.5687	.4225	.2359
3	.9539	.9410	.8963	.8495	.7857	.7006	.5863	.4378	.2457
2	.9613	.9530	.9081	.8636	.8017	.7173	.6036	.4529	.2555

TABLE 2: KING WITH KICKER

I'm going to use the language of domination, that is, I'll say that a hand K-x is dominated by any opponent hand containing either an ace or K-y, where y is of bigger rank than x. We use the obvious analogous terminology for Q-x. Table 2 (Table 1 is in Part 1 of this two-part series) gives the probabilities that at least one opponent has a hand dominating K-x for various ranks x and varying numbers of opponents. The rows are labelled for the rank of the kicker x held by our given player. The columns headed by numerals correspond to the number of opponents our player is facing.

For example, if you are holding K-9 with five opponents, then you look across the row labelled 9 until you reach the column headed by 5. The entry you find is .6744. This is the probability that one or more of your opponents has your hand dominated. In other words, in six-handed hold'em, two out of three times you find K-9 you are dominated by at least one opponent hand.

An important point that must be emphasized is that these are exact probabilities for the cards that your opponents have been dealt. Just because you are dominated does not mean that subsequent betting action, as well as the board cards, will not allow you to gain the upper hand. The value in looking at the

probabilities in Tables 2 and 3 is that it might sober a few players enthusiasm for certain hands containing a lone K or Q.

Following this line of thought, look at K-J. Against just a single opponent the probability of being dominated is .1673. This is essentially 1/6 so that in heads-up play, K-J is dominated about one out of every six times. I don't know whether this agrees with most people's intuition, but I suspect many players would guess it is not dominated that frequently.

Scanning across the appropriate row from right to left, you will notice that the probability first reaches more than .5 with just four players. Ask some of your poker playing friends how many opponents they think are required before the chances are greater than 1/2 that K-J is dominated by at least one hand. I speculate that most people will make a guess that is bigger than four. Finally, take a look at the entry for nine opponents. The probability is .8637 so that about six out of every seven times a player is holding K-J, the hand is dominated in a ten-handed hold'em game.

kicker	5	4	3	2	1
J	.8593	.7832	.6733	.5109	.2996
10	.8690	.7953	.6870	.5244	.3094
9	.8784	.8070	.7004	.5378	.3192
8	.8874	.8183	.7135	.5510	.3290
7	.8960	.8293	.7263	.5640	.3388
6	.9042	.8399	.7388	.5769	.3486
5	.9120	.8501	.7511	.5897	.3584
4	.9195	.8600	.7631	.6024	.3682
3	.9265	.8695	.7748	.6149	.3780
2	.9333	.8787	.7863	.6272	.3878

TABLE 3: QUEEN WITH KICKER

I am including a table for Q-x so that people who are interested in developing a feel for the initial strengths of hands may do so for Q-x. I myself find these numbers interesting. Even a hand like Q-J is dominated more than half of the time with only two opponents. Then look at how bad the situation is with five opponents for any Q-x. I'll say again that your play of the hand is extremely important, but it is valuable to have some insight into the comparitive strengths of beginning hands.

I'll conclude this article with a few comments about the derivation of the probabilities for Parts 1 and 2. As mentioned in Part 1, I used a straightforward application of inclusion-exclusion. That approach works easily because there are at most three players who can dominate an A-x hand. This means that one needs to pursue the inclusion-exclusion to a depth of three. In contrast, if one attempts to use inclusion-exclusion for a hand like K-x, you are opening yourself up to a computational nightmare. The reason is that there may be as many as seven hands that dominate K-x. This means that theoretically you might have to go to depth seven in the inclusion-exclusion process. I say "theoretically" because in many cases inclusion-exclusion converges so quickly that you can get

a very accurate approximation after depth three or four. However, this is one problem for which inclusion-exclusion does not converge quickly. Consequently, this is a problem for which it is much simpler to calculate the probability that none of the opponent hands dominate the given hand. This is the method I used throughout Part 2. Anyone interested in the details will find them at my website.