Going Low: part 2

Brian Alspach

I presented some basic information about the chances of being dealt qualifying low hands in 7-card stud high-low with an 8-low qualifier in Part 1. Looking at the chances for various kinds of low hands, as given in the table in Part 1, could lead to the belief that low hands are not worth the trouble. However, there is one extremely important fact to remember about 7-card stud; namely, a player is not faced with decisions until three cards have been dealt. So we should be far more interested in the chances of completing hands. That is the subject of this article.

We first consider the comparison between starting with one, two and three low cards. We do not, of course, consider starting with no low cards because it is impossible to make a low in this situation with only four cards to come. There are many possibilities and because we are seeking only a rough comparison at this stage, we shall make the assumption that there are seven opponents none of whose upcards have a rank that could be used for completing our starting hand to an 8-low or better.

Let me say a few words about obtaining the next set of probabilities. A given player knows her own three cards and the seven upcards of her opponents. Thus, she is drawing her remaining four cards from 42 unseen cards. This means there are $C(42, 4) = 111,930$ possible completions. All we do now is count the number of completions that give her an 8-low or better. We then divide by 111,930 to get the probability. The information is listed in the next table.

low cards	completions	probability of low	approximate odds against
3	66,440	.5936	2-to-3
2	32,640	.2916	7-to-3
1	2,240	.02	49-to-1

Let me make one fact clear about the preceding table. The number of low cards refers to the number of distinct low ranks in the player's hand. So a player who has been dealt A-A-3 has two low cards. One dramatic figure that leaps out of the table is the probability of making a low when holding only one low rank. The chances are so poor that a player should forget about low in this case. She should continue with the hand only if there is some other compelling reason such as being dealt a hand like 2-2-2. In this case the player stays in because she likely will win high. Once in a great while, a low will come via the back door.

We now concentrate on hands with a reasonable chance of making a low. Suppose we have been dealt three distinct low ranks. There are many possible patterns for our seven opponents to hold low cards that we need. Thus, we'll look at a few typical patterns. The probability that we'll complete a qualifying low is:

- .5936 when none of the cards we need is showing,

- .554 when one card we need is showing,
- .5046 when three cards of distinct ranks we need are showing, and
- .3993 when five cards, with one pair, we need are showing.

Note that the probability of making a low, when starting with three distinct low ranks and five cards you need showing, is substantially better than starting with two low ranks and none of the cards you need showing. This demonstrates how much more powerful three good beginning cards are.

Now consider hands with three distinct low ranks after the fourth card has been dealt. This corresponds to either starting with three low cards and missing on fourth street, or starting with two low cards and catching another one on fourth street. We now are going to assume that four of your opponents have folded before fourth street so that the player knows 14 of the cards. The probability that we'll complete a qualifying low is:

- .4372 when one card you need is gone,
- .3658 when three cards of distinct ranks you need are gone,
- .2958 when five cards, including one pair, you need are gone,
- .2285 when seven cards, including two pairs, you need are gone.

The preceding numbers give one a good idea of what the chances are of completing a low when holding three distinct low ranks after fourth street for any situation by making reasonable extrapolations. If you hold four distinct low ranks, it is easy to determine the chances of making a low. There are 16 cards that would complete a qualifying low hand. Simply subtract the number of them you have seen from 16 obtaining a number m. Let n be the total number of unseen cards. The probability of catching a low on the next card is m/n and the odds against are $(n - m)$-to-m.