There were several unfortunate typos in the article as printed in the magazine. They have been corrected in this file.

Facts about flopping pairs in hold'em

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This article is motivated by some e-mail correspondence with Tim Newsham regarding probabilities for opponents flopping bigger pairs. I thank Tim for his question.

Before getting to Tim's questions, let's start at the beginning with the basic probability distribution for clumping hands (ignoring straights and flushes) for a player holding two cards of distinct ranks. The next table provides this information and is valid no matter how many players are in the game.

quads	full house	trips	two pairs	one pair	none of preceding
.0001	.00092	.01571	.0404	.4041	.5388

Probabilities for Player Holding Two Cards of Distinct Ranks

Let's say a few words about how these numbers are determined. The player P is holding two cards x,y of different ranks. This means there are $C(50,3) = 19,600$ possible flops for P. As an example, here is how we get the probability for flopping two pairs. There are two ways P can flop two pairs: The flop is either of the form x,y,z, or P flops a card matching one of x or y together with another pair.

For the first type, there are three choices for each of the ranks x and y, and there are 44 choices for the remaining card. Multiplying gives 396 flops of this type. For the second type, there are six choices for a card of rank x or y. There are 11 choices for the rank of the pair and six choices for a pair of that rank. Multiplying gives 396 flops of this type. Adding the two numbers produces 792 flops that give P two pairs. Dividing by 19,600 yields the probability .0404 as given in the table.

Now move to a player holding a pocket pair. The next table gives the probabilities for the various flops. Again we are ignoring straights and flushes. There are still 19,600 flops, but now the possible outcomes and the probabilities change. The entries are derived in a manner similar to those used for the table above.

quads	full house	trips	two pairs	one pair
.00245	.0098	.1078	.1616	.7184

Probabilities for Player Holding a Pair

We now change our viewpoint and consider what flops with cards of three distinct ranks may have done for players who have seen the flop. Suppose the flop has come with cards of ranks $x < y < z$, where the inequality means that z is the largest rank, y is the next largest rank and x is the smallest rank. If m random hands have been dealt, then the probability that someone has flopped a pair or better (ignoring sequential hands) is shown in the next table.

Rank	Number of Players								
	2	3	4	5	6	7	8	9	10
z	.2298	.3302	.4214	.5040	.5783	.6448	.7039	.7560	.8017
y, z	.4175	.5640	.6785	.7667	.8338	.8839	.9208	.9473	.9660
x, y, z	.5687	.7255	.8295	.8969	.9394	.9656	.9812	.9902	.9951

Number of Players and Flopping Pair or Better

Let's discuss reading the preceding table. The column headed "Rank" indicates the ranks for the flopped pair or better. The row for z gives the probabilities that one or more players has flopped a pair or better of rank z, where the number of players is indicated by the column heading. The row for y, z indicates the probabilities one or more players has flopped a pair or better of ranks y or z. Finally, the row for x, y, z gives the probabilities that one or more players has flopped a pair or better of one of the ranks.

Finally, we consider flops in relation to a given player's hand. The next table pertains to how many ranks in the flop are larger than the rank of a pair our given player has been dealt, or has made, when either of these happen. If our given player has not made a pair, then we consider all three ranks.

Rank	Number of Opponents								
	2	3	4	5	6	7	8	9	10
z	.2389	.3426	.4364	.5208	.5964	.6635	.7228	.7747	.8196
y, z	.4322	.5812	.6961	.7835	.8489	.8968	.9314	.9558	.9724
x, y, z	.5862	.7429	.8445	.9087	.9482	.9717	.9852	.9927	.9966

Number of Opponents and Flopping Higher Pair

Just to make certain that you know how to read the preceding table, let's look at two examples. First, suppose our given player has the hand 7-7 and the flop has come 3-9-J. There are two larger ranks in the flop so that the appropriate row of the table is the y, z row. If there are 4 opponents, one goes to the column headed by "4" and finds there is a probability of .6961 that at least one of them has flopped a pair, or better, of either rank 9 or rank J.

Second, suppose our player has seen the flop with 10-9 and the flop has come 4-9-K. She has flopped middle pair so that the flop contains only one larger rank. If she has 6 opponents, she goes along the z row until reaching the column headed by "6". She finds a probability of .5964 that at least one player has flopped a pair, or better, of kings.

It is important to remember that we have made the assumption the opponent hands are random. How much players' decisions to stay in affect the numbers is a topic for another discussion.