Is positive expectation all it's cracked up to be?

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My last two articles dealt with Bib Ladder's question regarding the point at which it becomes profitable to play at the online hold'em bad-beat jackpot tables. His question tacitly accepts the premise that when a bet has positive expectation, a player enjoys an edge and should make the bet.

Indeed, the long term prognosis for a player making bets for which he has positive expectation is just what it says; the player has an excellent chance of coming out ahead in the long run. On the other hand, there is a potential problem in that the long run may exceed useful bounds on our time. Government run lotteries, Caribbean stud jackpots, and bad-beat poker jackpots have the common feature of escalating jackpots, with a low probability of occurrence, that eventually become sufficiently large so that a player has a positive expectation by playing.

Is it realistic to start playing one of these games because of a large jackpot of some kind giving a player a theoretical positive expectation? Let's look at the bonus bet in Caribbean stud because of the simplicity of analyzing it. I'll use payouts of $50 for a flush, $100 for a full house, $500 for four-of-a-kind, 10% of the jackpot for a straight flush, and 100% of the jackpot for a royal flush. It turns out that the break-even point for this set of payouts is about $226,260. So what can you expect if you start playing a Caribbean stud game, with these payouts, when the jackpot surpasses $226,260?

The contribution towards the player's expectation for the bonus bet coming from flushes, full houses, and quads is about 36% of the total expectation. In other words, if the two straight flush bonuses were removed, the player would be contributing about 64 cents of every dollar he bets to the house. The two straight flush bonuses form the bulk of the player's positive expectation and contribute about equally towards it. So the bet is still very negative if the royal flush bonus is not included. Thus, one needs to play for a royal flush in order to realize the advantage of the positive expectation. So let's see what the long run means for Caribbean stud.

There are four royal flushes in a standard 52-card deck. Thus, the probability of hitting one is 1/649,740. This means that if you play 10,000 hands of Caribbean stud, you have about a 1 in 65 chance of hitting a royal flush. If you play 100,000 hands of Caribbean stud, you have about a 1 in 7 chance of hitting a royal flush. Even a 1 in 7 chance is not that great.

A Caribbean stud table with a shuffling machine can do around 45 hands an hour. It then would take a player about 2,222 hours of play to log 100,000 hands. Playing four hours of Caribbean stud every day would require about 555 days to play the 100,000 hands. So the difficulty with this strategy is that the positive expectation comes about because of a low probability event, and the length of time required to give one a reasonable shot at the low probability

event is very long. Most players playing because of the big jackpot, in fact, will be donating to the casino at an exorbitant rate.

The situation for a lottery such as the 6/49 lottery is also not at all encouraging. Some people have proven the existence of a winning lottery strategy, but it depends on assumptions that are difficult to verify. The essential idea is that when the jackpot is sufficiently large, a player must pick six numbers that greatly increase his chances of being the only winner should the six numbers miraculously be chosen. (This can be a problem. I was visiting New Zealand once when 21 different tickets all had the same winning six numbers. I suspect some people may have made some poor decisions about their futures in the initial exuberance of learning they had a winner.) The impracticality of the lottery strategy is that hundreds of years are involved in carrying out the strategy, and the difficulty comes from having to choose a six-number ticket that is unpopular. So it seems to me that worrying about playing the lottery when it offers positive expectation really is a red herring. If you enjoy playing the lottery, just do it.

There is an important difference between lotteries and Caribbean stud, and bad-beat jackpot poker tables. For the first two, the other players are essentially irrelevant (although someone choosing the same six numbers in the lottery has an effect). At the poker table, the other players may be affected by the jackpot. The same problem occurs regarding the time frame required to realize a positive expectation, so ignore the bad-beat jackpot (unless it really becomes a factor) and act as if your table will never see it. Instead, figure it is costing you an extra five cents a hand to play at the table. Then determine if other players are making mistakes they might not ordinarily make, that are worth more than five cents a hand. If so, you actually may be gaining by being at the table because of game conditions. If the jackpot materializes, that is just frosting on the cake.