Due: Monday, November 14th (9:30 a.m.)

Reading

From the textbook: Section 4.4.4. Chapter 7 through Section 7.2.1. Sections 2.7 and 7.4, briefly skim 7.5.

Chapter 3 of Applegate, Bixby, Chvátal and Cook, which tells a little bit about the development of the ideas that we are covering.

Problems for Math 408 and Math 708

All problems to be submitted via Canvas. Please submit a single file names `hw4.pdf` containing all your written work, along with files `hw4.dat` and `hw4.mod` for the AMPL question (question 4); for this exercise separate files for part (b) would be helpful, which you can call `hw4b.dat` and `hw4b.mod`. Please make sure to write your name on the first page of `hw4.pdf` and in the comments of `AMPL` files.

1. Consider the personal knapsack problem you made in the first homework assignment.

a. Let S be the set of 0-1 points feasible for your knapsack problem. What dimension is the face of conv(S) defined by your personal knapsack inequality? If the right hand side of your personal knapsack inequality is fractional, consider also the inequality obtained by rounding down the fractional part on the right. This will also be a face of conv(S). What is its dimension?

b. Use your personal knapsack inequality to derive 3 minimal cover inequalities for conv(S). (If the LP relaxation to your personal knapsack problem gives an integer solution, work with the amended problem as in previous assignments.)

c. Solve the tightened problem given by adding the minimal cover inequalities to your basic knapsack inequality or its rounded version (from part a.) if the right hand side is fractional.

2. Describe all the faces of the 4-dimensional cube. How many are there in total?

3. Find five extreme points of the convex hull of the set $\{x \leq \sqrt{2}y, \;\; x \geq 1, \;\; x, y \in \mathbb{Z}_+\}$.

4. Consider the *cut polytope* of Example 3.36 in the text. Show that it is full-dimensional for $n = 2, 3, 4$.

5. Consider the following 0-1 knapsack polyhedron:

$$X = \{\mathbf{x} \in \{0,1\}^6 \mid 5x_1 + 3x_2 + 8x_3 + 9x_4 + 11x_5 + 8x_6 \leq 14\}.$$

a. What is the cover inequality corresponding to variables $\{1, 2, 3\}$?

b. What is the dimension of the face of $P_I = \text{conv}(X)$ represented by this cover inequality?

c. ~~Lift the inequality you found in part (1) in variable 5, and then lift the resulting inequality in variable 6.~~

Additional Problems for Math 708

6. Textbook Exercise 3.3.

7. A system of linear inequalities $\{A\mathbf{x} \leq \mathbf{b}\}$ is <u>totally dual integral (TDI)</u> if for all $\mathbf{c} \in \mathbb{Z}^n$ such that $\{\max \mathbf{c}^t\mathbf{x} \mid A\mathbf{x} \leq \mathbf{b}\}$ has a finite optimum value, the dual linear program $\{\min \mathbf{b}^t\mathbf{y} \mid A^t\mathbf{y} = \mathbf{c}, \mathbf{y} \geq \mathbf{0}\}$ has an integer optimum.

Show that the system $\{(x, y) \in \mathbb{R}^2 \mid x + y \leq 0, x - y \leq 0\}$ is not TDI, but that if we add the redundant inequality $x \leq 0$, the system becomes TDI.

Comments: In fact, if $\{A\mathbf{x} \leq \mathbf{b}\}$ is TDI, then P is the convex hull of S. If A is TUM, then $\{A\mathbf{x} \leq \mathbf{b}\}$ is TDI for *any* $\mathbf{b} \in \mathbb{Z}^n$.