MATH 408 AND 708 ASSIGNMENT #2

Due: Wednesday, October 7th (noon)

Reading

From the textbook, Sections 3.3 (should be review), 4.1, 4.2 and the introduction to Section 4.3.

Chapters 3 and 4 of the AMPL book, available at:

`http://ampl.com/resources/the-ampl-book/chapter-downloads/`

The remainder of Chapter 1 of Applegate, Bixby, Chvátal and Cook.

Problems for Math 408 and Math 708

All problems to be submitted via Canvas. Please submit a single file names `hw2.pdf` containing all your written work, along with files `hw2.dat` and `hw2.mod` for the AMPL question (question 3). Please make sure to write your name on the first page of `hw2.pdf` and in the comments of `hw2.dat` and `hw2.mod`.

1. Consider the personal knapsack problem that you made in the first homework assignment. Look at the LP relaxation from part 1 b. If the LP relaxation is an integer vector, then before proceeding you will add 1 to the right hand side of your knapsack constraint and work with your *amended personal knapsack problem* instead of the original problem that you were working with.

At this point you should be working with a problem whose LP relaxation is not integral. Perform one step of branch-and-bound on this problem. That is, branch on a non-integer variable to get two subproblems. For each of the two subproblems:

a. Construct a primal (lower) bound for the optimum using the following greedy algorithm: begin with the feasible point $\vec{x} = 0$. For each i from 1 to 9 in turn, see if the point remains feasible if x_i is set to 1. If it is, then set keep $x_i = 1$. Otherwise, reset to $x_i = 0$.

b. Construct a dual (upper) bound for the optimum by solving the LP relaxation of the problem, where $\vec{0} \leq \vec{x} \leq \vec{1}$.

c. Find the optimal (integer) solution to the subproblem using AMPL. Compare this to the two bounds you have found for that subproblem. You do not submit the AMPL files.

2. Solve your [amended] personal knapsack problem using branch-and-bound. You don't have to give full details, but draw the branch-and-bound tree as is done in Figure 1.4 in the textbook.

Note that you can use the dual bound from question 1a. to help prune the tree. If you have expanded at least 20 nodes without verifying optimality, that is sufficient – you can stop there.

3. Exercise 20-1 from the AMPL book. (It references Exercise 1-1, but doesn't require it.) Submit the only the AMPL files for part (b), you do not submit the modified files related to the relaxation.

4. Textbook Exercise 2.1.

5. Give an example of a $\{-1, 0, 1\}$ matrix A and an integer vector b such that the set $\{Ax \leq b \mid x \in \mathbb{R}^n\}$ is an integer polytope, but A is not totally unimodular.

Additional Problems for Math 708

6. Consider the `crucipixel` game of textbook exercise 2.30.

   a. Formulate the particular 10×10 example given in the problem as a 0-1 linear program.

   b. Solve the problem with AMPL, submitting files as in Problem 3, but with names `grad2.dat` and `grad2.mod`.

   c. Explain how to generalize the formulation from part a. to an arbitrary $m \times n$ `crucipixel` game.

7. Textbook Exercise 4.7.

8. Consider modelling a scheduling problem where machine can be switched on at most k times, with discrete time segments indexed by t. This problem can be modelled using variables y_t representing whether the machine is on during period t, and z_t which representing whether the switching on happened during period t. This can be formulated via the following inequalities:

$$y_t - y_{t-1} \leq z_t \leq y_t \text{ for all } t; \qquad \sum_t z_t \leq k; \qquad 0 \leq y_t, z_t \leq 1 \text{ for all } t.$$

Show that the matrix encoding these constraints is totally unimodular.

Graduate Student Projects

Math 708 students will give presentations surveying additional topics in integer programming. These presentations will take place in the final full week of class, on Wednesday, December 2nd, Friday, December 4th and Monday, December 7th. Presentations will last for 20 minutes, followed by 5 minutes for questions.

Given the on-line format of the course, well prepared slides will be essential.

Possible sources for topics include sections of the textbook not covered in class, and the book *Algebraic and Geometric Ideas in the Theory of Discrete Optimization* by De Loera, Hemmecke and Köppe or the surveys from *50 Years of Integer Programming 1958-2008* by Jünger. Other topics may be possible, please see the instructor if you have something else in mind. It may be helpful to discuss with your advisor which topics are relevant to your research. Please sign-up for a date and topic. First-come, first-served.

Students who are interested in doing a more extensive and in-depth presentation may consider presenting a 45 minute talk in the Operations Research Seminar on Thursdays (most likely November 26th or December 3rd) at 2:30 p.m. subject to availability and consent of the instructor.