Due: Friday, November 9th (in class)

Reading

From the textbook: Section 5.2.4.

Section 4.4.4.

Chapter 7 through Section 7.2.1.

Sections 2.7 and 7.4, briefly skim 7.5.

Chapter 8 through Section 8.2

(Optional) Applegate, Bixby, Chvátal and Cook, Chapter 5 through Section 5.5. This covers several of the topics we have been working on in the context of the TSP. Chapters 3 and 4 are interesting too.

Problems for Math 408 and Math 708

1. Consider the personal knapsack problem you made in the first homework assignment.

   a. Let S be the set of 0-1 points feasible for your knapsack problem. What dimension is the face of conv(S) defined by your personal knapsack inequality.

   b. Use your personal knapsack inequality to derive 3 minimal cover inequalities for conv(S).

2. Describe all the faces of the 4-dimensional cube. How many are there in total?

3. Consider the *cut polytope* of Example 3.36 in the text. Show that it is full-dimensional for $n = 2, 3, 4$.

4. Exercise 20.4 (a) and (b) from the AMPL book, available at:

`http://ampl.com/resources/the-ampl-book/chapter-downloads/`.

Please submit your answer to problem 4 directly to the teaching assistant by e-mail (`xla97` at `sfu` dot `ca`). All file names should begin: `math_408_1187_name_hw4_q4` where `name` is your family name. Submit the relevant .dat, .mod and a .pdf file showing your output in a single e-mail. If you would like to submit additional questions via e-mail, you may do so, but only if they are *typeset*. (Do **not** include documents that are produced by a scanner.) If you wish to do this, include your remaining answers either in a single .pdf file named: `math_408_1187_name_hw4_qall`, or with one .pdf file per question names `math_408_1187_name_hw4_q1` (etc., as appropriate). Here again, `name` is substituted with your own family name.

5. Consider the following 0-1 knapsack polyhedron:

$$X = \{\mathbf{x} \in \{0,1\}^6 \mid 5x_1 + 3x_2 + 8x_3 + 9x_4 + 13x_5 + 8x_6 \leq 15\}.$$

   a. What is the cover inequality corresponding to variables $\{1, 2, 3\}$?

   b. What is the dimension of the face of $P_I = \text{conv}(X)$ represented by this cover inequality?

   c. Lift the inequality you found in part (1) in variable 5, and then lift the resulting inequality in variable 6.

Additional Problems for Math 708

6. Textbook Exercise 3.3.

7. Exercise 20.4 (c) and (d) from the AMPL book. Submissions should follow the format of problem 4.

8. A system of linear inequalities $\{A\mathbf{x} \leq \mathbf{b}\}$ is *totally dual integral (TDI)* if for all $\mathbf{c} \in \mathbb{Z}^n$ such that $\{\max \mathbf{c}^t\mathbf{x} \mid A\mathbf{x} \leq \mathbf{b}\}$ has a finite optimum value, the dual linear program $\{\min \mathbf{b}^t\mathbf{y} \mid A^t\mathbf{y} = \mathbf{c}, \mathbf{y} \geq \mathbf{0}\}$ has an integer optimum.

Show that the system $\{(x, y) \in \mathbb{R}^2 \mid x + y \leq 0, x - y \leq 0\}$ is not TDI, but that if we add the redundant inequality $x \leq 0$, the system becomes TDI.

Comments: In fact, if $\{A\mathbf{x} \leq \mathbf{b}\}$ is TDI, then P is the convex hull of S. If A is TUM, then $\{A\mathbf{x} \leq \mathbf{b}\}$ is TDI for *any* $\mathbf{b} \in \mathbb{Z}^n$.

Tentative schedule of graduate presentations

Each graduate student will present a brief introductory lecture on an additional topic in integer programming. This should contain substantial mathematical content and be understandable to the undergraduate students. The talks will be 20 minutes, followed by a 5 minute question period. Overheads will be submitted as part of the grading.

These talks will take place during the week of November 26th to 30th. The tentative schedule and topics are as follows:

Monday, November 26th **Jasdeep Dhahan**, Fourier Elimination (Section 3.1 in the text).

Wednesday, November 28th (early) **Chloe Li**, topic *to be determined*.

Wednesday, November 28th (late) **Navpreet Kaur**, Weekly Scheduling Modules for Traveling Therapists (Socio-Economic Planning Sciences 47 (3) 2013, pp. 191-204).

Friday, November 30th (early) **Haggai Liu**, topic *to be determined* (from Section 1.5 in the text).

Friday, November 30th (late) **Shawn Yan**, Graver Bases (Sections 3.1 and 3.2 in De Loera, Hemmecke and Köppe).

Please let me know about any possible errors or adjustments in this schedule.