Due: Friday, October 19th (in class)

Reminders

The midterm will take place in class on Friday, October 26st. It will cover material from class up to Wednesday, October 24th.

Math 708 students must select a presentation topic and a date for the presentation. Please consult me if you have not done this. To give you some idea of what might be suitable, topics from the previous edition of the course are listed at the bottom of this assignment.

Reading

From the textbook, Sections 5.2.4, and 3.4 through 3.9.

Problems for Math 408 and Math 708

1. For each of the following sets, find a valid inequality cutting off the given fractional point:

   a. $\{(x_1, x_2) \in \mathbb{Z}^2_+ \mid x_1 \leq 5, \;\; x_1 \leq 4x_2\}$ $\qquad (x_1^*, x_2^*) = (5, \frac{5}{4})$.

   b. $\{(x_1, x_2, x_3) \in \mathbb{Z}^3_+ \mid x_1 + x_2 - 2x_3 \leq 0, \;\; x_1, x_2, x_3 \leq 1\}$ $\qquad (x_1^*, x_2^*, x_3^*) = (1, 0, \frac{1}{2})$.

   c. $\{(x_1, x_2, x_3, x_4) \in \mathbb{Z}^4_+ \mid 4x_1 + 8x_2 + 7x_3 + 5x_4 \leq 33\}$ $\qquad (x_1^*, x_2^*, x_3^*, x_4^*) = (0, 0, \frac{33}{7}, 0)$.

You should explain how you know that the inequality is valid.

2. Consider the integer programming problem:

$$(IP) \qquad \begin{array}{ll} \text{maximize} & 4x_1 - 6x_2 \\ \text{subject to} & x_1 + x_2 \leq 5, \\ & 2x_1 - 3x_2 \leq 1, \\ & \quad x_1, x_2 \geq 0; \qquad x_1, x_2 \in \mathbb{Z} \end{array}$$

   a. Put the system in standard (equality) form.

   b. Use the simplex method to find a point x^* maximizing the linear programming relaxation of this problem.

   c. Use the optimal basis that you have found to generate a Gomory cut, that is, an inequality cutting x^* but no feasible points of (IP).

3. Exercise 20.3 (a) and (b) from the AMPL book, available at:

`http://ampl.com/resources/the-ampl-book/chapter-downloads/`.

Please submit your answer to problem 4 directly to the teaching assistant by e-mail (`xla97` at `sfu` dot `ca`). All file names should begin: `math_408_1187_hw3_name_q3` where `name` is your family name. Submit the relevant .dat, .mod and a .pdf file showing your output in a single e-mail. If you would like to submit additional questions via e-mail, you may do so, but only if they are *typeset*. (Do **not** include documents that are produced by a scanner.) If you wish to do this, include your remaining answers either in a single .pdf file named: `math_408_1187_hw3_name_qall`, or with one .pdf file per question names `math_408_1187_hw3_name_q1` (or `q2`, `q4`, `q5`, `q6`, `q7`, `q8` as appropriate). Here again, `name` is substituted with your own family name.

4. a. Prove that the intersection of two convex sets is a convex set.

   b. Show by example that the union of two convex sets may not be a convex set.

   c. Give an example of two *disjoint* convex sets whose union is a convex set.

5. Prove that if a cone is pointed, then it only has one extreme point, namely $\vec{0}$.

Additional Problems for Math 708

6. Recall the *Set Covering Problem* (SCP): have objects $\{1, 2, \ldots, n\}$, and sets $\{S_1, \ldots, S_m\}$ of objects with *costs* $c_1, \ldots, c_m$ attached to the sets. The goal is to find the cheapest collection of sets covering all the objects.

A related decision problem is *Budget Set Covering Problem* (BSCP) where additionally you are given a budget b, and the problem is to determine if there is a collection of sets covering the objects with total cost at least b. Explain how an algorithm for (BSCP) can be used to solve (SCP). Can this be done in polynomial time in the input size of the (SCP)?

7. Consider the problem of finding a maximum stable set of a graph (a maximum set of vertices with no two vertices sharing an edge). Formulate this problem as:

$$\max \sum_{v \in V} x_v \quad \text{subject to} \quad x_{v_1} + x_{v_2} \leq 1 \quad \forall (v_1, v_2) \in E \quad \text{and} \quad x \in \{0,1\}^{|V|}$$

Show that for any complete subgraph (*clique*) W of G, you can obtain the clique inequality $\sum_{v \in W} x_v \leq 1$ by repeatedly applying rounding cuts.

8. Textbook Exercise 3.7.

Examples of project topics from the previous offering of Math 708

Solving linear Diophantine equations (Chapter 2, Sections 2.1-2.3 in De Loera, Hemmecke and Köppe).

Presolve algorithms.

Gröbner bases (Section 7.1 in Bertsimas and Weismantel).

Robust discrete optimization (Section 14.1 in Bertsimas and Weismantel).