Due: Friday, October 7th (in class)

Reading

From the textbook, Section 3.1, Appendix B, Section 9.1, Section 9.2 and Section 11.1.

(Optional) Chapter 2 of Applegate, Bixby, Chvátal and Cook, which shows several interesting problems that can be modelled as a TSP, and hence as an integer program.

Problems for Math 408 and Math 708

1. Consider the personal knapsack you made in the first homework assignment.

a. Construct a primal (lower) bound for the optimum using the following greedy algorithm: begin with the feasible point $\vec{x} = 0$. For each i from 1 to 9 in turn, see if the point remains feasible if x_i is set to 1. If it is, then set keep $x_i = 1$. Otherwise, reset to $x_i = 0$.

b. Construct a dual (upper) bound for the optimum by solving the LP relaxation of the problem.

c. Compare the two bounds to the optimum you found in the first homework assignment, by writing a simple inequality of the form $l \leq opt \leq u$.

2. Exercise 4.4 from the AMPL book, available at:

`http://ampl.com/resources/the-ampl-book/chapter-downloads/`.

The files used to produce your final solutions should be collected in an electronic archive (e.g. a .zip or .gz file) and e-mailed directly to him at `arafiey` at `sfu.ca`. Use files names of the form `hw2-2-stephen.dat`, replacing "stephen" with your own surname and extensions as appropriate. For the discussion parts of the question, either include with the electronic archive as a .pdf file (use filename `hw-2-2-stephen-discussion.pdf`, but with your own surname), or include it with your written solutions. The discussion should contain the results of the problems.

3. Given a graph $G = (V, E)$, formulate the problem of finding the largest stable set in G as an integer linear program. A *stable set* is a set $U \subseteq V$ of vertices that do not share any edges in common. Check that the dual of the relaxation of the linear program that you wrote down is the linear program used in the formulation of the problem of finding the minimum cover of the nodes by *cliques*, which are subgraphs where every pair of vertices are connected by an edge. Finally, illustrate optimal integer solutions to both problems on the graph of Figure 1

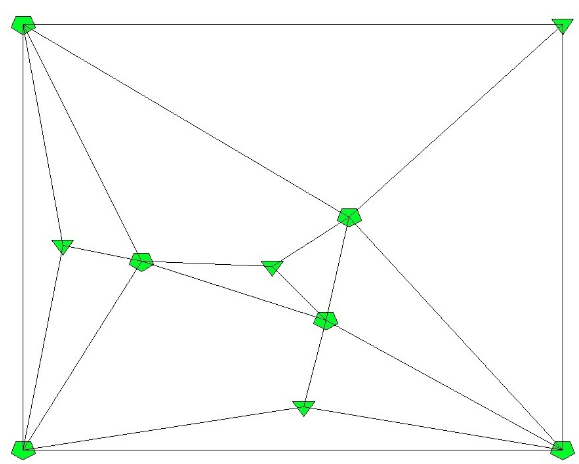

Figure 1: Graph for problem 2. `Image credit: James Preen via stackexchange`

4. Give an example of a $\{-1, 0, 1\}$ matrix A and an integer vector b such that the set $\{Ax \leq b \mid x \in \mathbb{R}^n\}$ is an integer polytope, but A is not totally unimodular.

5. Textbook Exercise 3.5.

Additional Problems for Math 708

6. Textbook Exercise 1.16.

7. Textbook Exercise 3.2.

8. Consider modelling a scheduling problem where machine can be switched on at most k times, with discrete time segments indexed by t. This problem can be modelled using variables y_t representing whether the machine is on during period t, and z_t which representing whether the

switching on happened during period t. This can be formulated via the following inequalities:

$$y_t - y_{t-1} \leq z_t \leq y_t \text{ for all } t; \qquad \sum_t z_t \leq k; \qquad 0 \leq y_t, z_t \leq 1 \text{ for all } t.$$

Show that the matrix encoding these constraints is totally unimodular.

Graduate student projects

Math 708 students will give presentations surveying additional topics in integer programming. These presentations will take place in the final full week of class, on Monday, November 28th, Wednesday, November 30th or Friday, December 2nd. Presentations will last for 20 minutes, followed by 5 minutes for questions.

Possible sources for topics include additional sections of the textbook, the book of de Lorea, Hemmecke and Köppe [1], or the current topic surveys in *50 years of integer programming* [2]. Other topics may be possible, please see the instructor if you have something else in mind. It may be helpful to discuss with your advisor which topics are relevant to your research. Please sign-up for a date and topic. First-come, first-served.

Students who are interested in doing a more extensive and in-depth presentation may consider presenting a 45 minute talk in the Operations Research Seminar on Thursdays at 2:30 subject to availability and consent of the instructor.

References

[1] Jesús A. De Loera, Raymond Hemmecke, and Matthias Köppe. *Algebraic and geometric ideas in the theory of discrete optimization*, volume 14 of *MOS-SIAM Series on Optimization*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA; Mathematical Optimization Society, Philadelphia, PA, 2013.

[2] Michael Jünger, Thomas Liebling, Denis Naddef, George Nemhauser, William Pulleyblank, Gerhard Reinelt, Giovanni Rinaldi, and Laurence Wolsey, editors. *50 years of integer programming 1958–2008*. Springer-Verlag, Berlin, 2010. From the early years to the state-of-the-art, Papers from the 12th Combinatorial Optimization Workshop (AUSSOIS 2008) held in Aussois, January 7–11, 2008.