Due: Monday, November 14th (in class)

Reading

Textbook pp. 185-6, Section 4.3, Section 4.4 up to the heading "Non-linear Optimization Problems", Section 1.4.

(Optional) Applegate, Bixby, Chvátal and Cook, Chapter 5 through Section 5.5. This covers several of the topics we have been working on in the context of the TSP. Chapters 3 and 4 are interesting too.

Problems for Math 408 and Math 708

1. Consider the set

$$X = \{(x, y) \in \mathbb{Z}_+^2 \mid 2x + 5y \leq 17,\ 7x + 3y \leq 23\}.$$

List and represent graphically the set of feasible points. Use this to find a minimal (facet) description of $\mathrm{conv}(X)$.

2. Textbook Exercise 2.1.

3. Textbook Exercise 2.2. For part b), conditions mean something like "all the a_i's are equal. In fact, this is a good start to getting one condition, you should figure out what remains to be said about b and λ. Finding these conditions and showing why they work is sufficient for full credit here; finding a second "different" condition can be a treated as a bonus.

4. Consider the following 0-1 knapsack polyhedron:

$$X = \{x \in B^6 \mid 5x_1 + 3x_2 + 8x_3 + 9x_4 + 13x_5 + 8x_6 \leq 15\}.$$

1. What is the cover inequality corresponding to variables $\{1, 2, 3\}$?
2. What is the dimension of the face of $P_I = \mathrm{conv}(X)$ represented by this cover inequality?
3. Lift the inequality you found in part (1) in variable 5, and then lift the resulting inequality in variable 6.

5. Recall the *stable set problem* from Homework 2. An *odd hole* in G is a cycle with an odd number of vertices and no additional edges between vertices of the cycle. If H is the an odd cycle, use the edge (2-clique) inequalities in your formulation and rounding to show that the following *odd hole* inequalities are valid for stable sets:

$$\sum_{j \in H} x_j \leq \frac{|H| - 1}{2}$$

Additional Problems for Math 708

6. Consider the stable set formulation from Example 2.6 in the textbook (without clique inequalities). Take the graph G which consists of a 5-cycle and a single vertex v_6 attached to each vertex of the cycle. Such graphs are sometimes called *wheels*. The 5-cycle inequality is valid for the 5-wheel.

1. What is the dimension of the face induced by the 5-cycle inequality? What is the dimension of the stable-set polytope of the 5-wheel?
2. Lift this face to a facet by adding a term representing the variable x_6 to the inequality.

7. Exercise 20.4 (a) through (d) from the AMPL book, available at:

`http://ampl.com/resources/the-ampl-book/chapter-downloads/`.

The files used to produce your final solutions should be collected in an electronic archive (e.g. a .zip or .gz file) and e-mailed directly to him at `arafiey` at `sfu.ca`. Use files names of the form `hw4-7-stephen.dat`, replacing "stephen" with your own surname and extensions as appropriate. Make sure to include your final solution as well as the input files.

8. Show that the system $\{x, y \in \mathbb{R}^2 \mid x+y \leq 0, x-y \leq 0\}$ is not TDI, but that if we add the redundant inequality $x \leq 0$, the system becomes TDI.

Tentative schedule of graduate presentations

Each graduate student will present a brief introductory lecture on an additional topic in integer programming. This should contain substantial mathematical content and be understandable to the undergraduate students. The talks will be 20 minutes, followed by a 5 minute question period. Overheads will be submitted as part of the grading.

These talks will take place during the week of November 28th to December 1st. The tentative schedule and topics are as follows:

Monday, November 28th (early) **Michael Friesen**, Solving linear Diophantine equations (Chapter 2, Sections 2.1-2.3 in De Loera, Hemmecke and Köppe).

Monday, November 28th (late) **Michelle Spencer**, Mixed Integer Programming topic *to be determined*.

Wednesday, November 30th (early) **Jingxin Guo**, Mixed Integer Programming topic *to be determined*.

Wednesday, November 30th (late) **Rimi Sharma**, Mixed Integer Programming in Portfolio Optimization details *to be determined*.

Friday, December 2nd (early) **Aniket Mane**, Gröbner bases (Section 7.1 in textbook).

Friday, December 2nd (late) **Bolong He**, Robust discrete optimization (Section 14.1), time permitting with applications to inventory theory (Section 14.4).

Please let me know about any possible errors or adjustments in this schedule.