Due: Friday, October 24th (in class)

## Reminders

The midterm will take place in class on Friday, October 17th. It will cover material from class up to Friday, October 10th.

Math 708 students must select a presentation topic and a date for the presentation. Please consult me if you have not done this.

## Reading

Chapters 9 and 10. Please carefully read Section 9.2.2, which provides the key definitions for the polyhedral approach, and Section 9.3, which uses the knapsack problem to illustrate the approach. You can skim the proofs in Section 9.2.3, and skip or skim Section 9.4.

## Problems for Math 408 and Math 708

1. Chapter 8 problem 2.

2. Consider the integer programming problem:

$$(IP) \qquad \begin{array}{ll} \text{maximize} & 4x_1 - 6x_2 \\ \text{subject to} & x_1 + x_2 \leq 5, \\ & 2x_1 - 3x_2 \leq 1, \\ & \quad x_1, x_2 \geq 0; \qquad x_1, x_2 \in \mathbb{Z} \end{array}$$

Find the point $x^*$ maximizing the linear programming relaxation of this problem, and find an inequality cutting $x^*$ but no feasible points of (IP).

3. Chapter 7 problem 1. There is a typo in the book: please replace 35 by 31 in the lower right. All upper bounds are assumed to come from feasible solutions.

4. Chapter 7 problem 3.

5. Prove that the intersection of any two faces of a polytope $P$ is also a face of $P$.

## Additional Problems for Math 708

6. Chapter 7 problem 5. The *assignment relaxation* of the TSP requires only that the number of edges entering and leaving each vertex is 1.

7. Consider the integer program

$$\min x_{n+1} \quad \text{subject to} \quad 2x_1 + 2x_2 + \ldots + 2x_n + x_{n+1} = n \quad \text{and} \quad x \in \{0,1\}^{n+1}$$

Prove that if $n$ is odd, a branch and bound algorithm (without using cuts) will have to examine at least $2^{\lfloor \frac{n}{2} \rfloor}$ candidate problems before it can solve the main problem.

8. Consider the problem of finding a maximum stable set of a graph (a maximum set of vertices with no two vertices sharing an edge). Formulate this problem as:

$$\max \sum_{v \in V} x_v \quad \text{subject to} \quad x_{v_1} + x_{v_2} \leq 1 \quad \forall (v_1, v_2) \in E \quad \text{and} \quad x \in \{0,1\}^{|V|}$$

Show that for any complete subgraph (*clique*) $W$ of $G$, you can obtain the clique inequality $\sum_{v \in W} x_v \leq 1$ by repeatedly applying rounding cuts.