Quiz: Friday, March 4th (in class)

Reminder

The second midterm will take place in class on **Wednesday, March 16th**.

Reading

For Monday, February 28th, Section 10.3.
For Wednesday, March 2nd, Section 10.4.
For Friday, March 4th, Section 11.1.
While we will not cover it in class, Section 10.6 contains interesting examples which help to motivate the study of recurrences, particularly for computer science students.

Assignment questions

Section 10.2: 1, 13, 24, 26.
Section 10.3: 1, 3.

Instructor question: In this exercise, we illustrate a classical approach to designing recurrences for counting sets of words with prescribed patterns.

Let $\mathcal{W}$ be the set of sequences on $\{0, 1, 2\}^*$ with no occurrence of 00 or 11. We want to design and solve a recurrence for w_n (the number of words in $\mathcal{W}$ of length n).

1. Let $\mathcal{A}$ be the subset of $\mathcal{W}$ composed of the sequences whose last symbol is a 0. Similarly $\mathcal{B}$ is the subset of sequences ending by 1 and $\mathcal{C}$ the subset of sequences ending by 2. If a_n (respectively b_n, c_n) is the cardinality of $\mathcal{A}_n$ (respectively $\mathcal{B}_n$, $\mathcal{B}_n$), prove that

$$w_n = a_n + b_n + c_n, \text{ for all } n \geq 1.$$

2. Hence, to find a recurrence for w_n we can try to define it using $a_n,\ b_n,\ c_n$, which we do now.

   First, prove the following statements:

   - For all $n \geq 1$, one has $a_n = b_n$.
   - For all $n \geq 2$, one has $a_n = a_{n-1} + c_{n-1}$.
   - For all $n \geq 1$, one has $c_n = w_{n-1}$.

   Next, using the equations above, show that

$$w_n = 2w_{n-1} + w_{n-2} \text{ for all } n \geq 3.$$

3. Compute w_1 and w_2, and solve the recurrence for w_n.

Some other questions worth trying

Section 10.1: 1, 3.
Section 10.2: 3, 9.