Quiz: Friday, January 21st (in class)

Reading

For Monday, Sections 3.5 and 3.6.
For Wednesday, Section 3.7.
For Friday, Section 8.1.

Assignment questions

Section 3.4: 8, 14.
Section 3.5: 4, 12.
Section 3.6: 4, 14, 24.
Instructor question: **Bayes' Theorem** can be expressed in the form:

$$Pr(E|F) = \frac{Pr(F|E)Pr(E)}{Pr(F)}.$$

The object of this question is to show how to use Bayes' Theorem to solve a simple "practical" problem.

We consider a disease that affects 0.1% of a given population. Researchers have developed a test for this disease that has the following properties:

1. when applied to a person with the disease, it will detect the disease with probability 0.99 (99% accuracy),
2. when applied to a healthy person, it will suggest this person has the disease (a false positive result) with probability 0.02 (false positive result on 2% of the healthy population).

We want to answer a few questions:

- What is the probability that someone selected at random tests positive for the disease ?
- What is the probability that someone who tests positive really has the disease ?

We consider the following sample space, composed of four events. We denote by D the event "having the disease" and by ND the event "not having the disease". We denote by pos the event "testing positive" and by neg the event "testing negative". The sample space contains four events $D.pos$ (a sick person tests positive), $D.neg$ (a sick person tests negative), $ND.pos$ (a healthy person tests positive) and $ND.neg$ (a healthy person tests negative).

You can follow the following steps:

1. Translate the properties of the problem regarding the disease and the test in terms of probabilities (possibly conditional probabilities).
2. Compute the probability of each of the possible events in the sample space.
3. Express the two probabilities we want to compute in terms of probabilities of events from the sample space.
4. Use the probability computation formulas and probability axioms seen in class and Bayes' Theorem to compute values for the probabilities obtained in point (c).

Some other questions worth trying

Section 3.4: 3, 7, 11.
Section 3.5: 1, 7, 15.
Section 3.6: 3, 5, 17.
Section 3.7: 1, 5.

Selected Hints & Answers

- 3.4, 8. $1/2$.
- 3.4, 14. (a) 0.08 (b) 0.29.
- 3.5, 4. $21/26$
- 3.5, 12. (i) $4/5$ (ii) $1/2$ (iii) $3/10$ (iv) $3/5$
- 3.6, 4. (a) $17/29$ (b) $5/17$
- 3.6, 14. 0.5
- 3.6, 24. (a) $1/4$ (b) $1/4$
- Instructor question. (a) There are 3 pieces of information to consider: 0.1% of the population has the disease, and properties (1) and (2). (b) use the formula for conditional probabilities. (c) One probability is conditional, one is not. (d) The probability that someone selected at random tests positive for the disease is 0.02097. The probability that someone who tests positive has really the disease is 0.0472. In other words, given the rarity of the disease, even if the test is positive, there is roughly a 5% chance that the disease is in fact present.