Fifth Homework Assignment for Math 408 and 708

Due: Friday, November 26th, 2010, in class.

Problems for Math 408 and 708:

1. Chapter 10 problem 1.
2. Chapter 11 problem 1.
3. Chapter 11 problems 2.
4. Using a mathematical software package such as MATLAB, generate 12 points in $\mathbb{R}^2$ by choosing each coordinate to be an integer uniformly at random in the range $[1, 100]$. Compute the matrix of pairwise distances between these points. (You can round the distances to the nearest integers.) Please include a computer printout showing the points and the pairwise distances.

   Apply the Christofides heuristic to find a good tour through these points. Draw a picture illustrating the points, the minimum spanning tree, the matching edges and the found tour. Can you improve this solutions by exchanging a pair of vertices?
5. Chapter 12, problem 2.

Additional problems for Math 708:

6. Chapter 10 problem 5.
7. Consider the assignment relaxation of the directed Travelling Salesman Problem (TSP):

$$\begin{array}{ll} \text{minimize} & \sum_{i,j} c_{ij} x_{ij} \\ \text{subject to} & \sum_i x_{ij} = 1 \text{ for } j = 1, \ldots, n \\ & \sum_j x_{ij} = 1 \text{ for } j = 1, \ldots, n \\ & 0 \le x_{ij} \le 1, x_{ij} \text{ integer for } i, j = 1, \ldots, n \end{array}$$

   We know that if we add subtour constraints to this description, we get an integer program formulating the TSP. Show that we can also get an integer programming formulation of the TSP by adding n additional variables $u_i, i = 1, \ldots n$ and additional constraints:

$$\begin{array}{l} u_1 = 1 \\ 2 \le u_i \le n \text{ for } i = 1, \ldots, n \\ u_i - u_j + 1 \le (n-1)(1 - x_{ij}) \text{ for } i, j \ne 1 \end{array}$$

   Remark: this program has polynomial size, but is much weaker than the subtour formulation.
8. Chapter 12, problem 4.

Reading:

Chapters 12 and 13 (the last one lightly).

Reminder:

The final exam is scheduled for **Thursday, December 9th at 8:30 a.m.**

Schedule of presentations:

Text references are to [4].

Friday, November 26th

(1) T.J. Yusun, 19-19.3, corner relaxations.
(2) Sara Taghipour, [1].
(3) Mehrnoush Malekesmaeili, 18-18.3, modelling combinatorial optimization problems with semidefinite programming.

Wednesday, December 1st.

(1) Farzana Sultana, 16, mixed integer programming computation.
(2) Yong Zhang, [3].

Friday, December 3rd.

(1) Ali Nadaf, [2].
(2) Piyashat Sripratak, 17.1-17.2, 17.7, symmetry in integer linear programming.
(3) Tanmay Deshpande, 15.4, polynomial optimization.
(4) Maria Tamayo, 15.4, polynomial optimization (continued).

Each presentation should be about 25 minutes. As part of the grading, students will submit a copy of the overheads used for the talk.

References

[1] Makoto Asano and Hiroshi Ohta. Single machine scheduling using dominance relation to minimize earliness subject to ready and due times. *International Journal of Production Economics*, 44(1-2):35–43, 1996.

[2] John T. Blake and Joan Donald. Mount sinai hospital uses integer programming to allocate operating room time. *Interfaces*, 32(2):63–73, 2002.

[3] Samuel Burer and Renato D. C. Monteiro. A projected gradient algorithm for solving the maxcut SDP relaxation. *Optim. Methods Softw.*, 15(3-4):175–200, 2001.

[4] Michael Jünger, Thomas Liebling, Denis Naddef, George Nemhauser, William Pulleyblank, Gerhard Reinelt, Giovanni Rinaldi, and Laurence Wolsey, editors. *50 years of integer programming 1958–2008*. Springer-Verlag, Berlin, 2010. From the early years to the state-of-the-art, Papers from the 12th Combinatorial Optimization Workshop (AUSSOIS 2008) held in Aussois, January 7–11, 2008.