Fourth Homework Assignment for Math 408 and 708

Due: Friday, November 12th, 2010, in class.

Problems for Math 408 and 708:

1. Chapter 8 problem 8.

2. Show that the system $\{x, y \in \mathbb{R}^2 | x + y \leq 0, x - y \leq 0\}$ is not TDI, but that if we add the redundant inequality $x \leq 0$, the system becomes TDI.

3. Chapter 9 problem 1.

4. Consider the following 0-1 knapsack polyhedron:

$$X = \{x \in B^6 | 7x_1 + 7x_2 + 7x_3 + 5x_4 + 13x_5 + 6x_6 \leq 19\}.$$

   (1) What is the cover inequality corresponding to variables $\{1, 2, 3\}$?
   (2) What is the dimension of the face of $P_I = \text{conv}(X)$ represented by this cover inequality?
   (3) Lift the inequality you found in part (1) in variable 5, and then lift the resulting inequality in variable 6.
   (4) Lift the inequality you found in part (1) in variable 6, and then lift the resulting inequality in variable 5.

5. Chapter 9 problem 3.

Additional problems for Math 708:

6. Consider the stable set formulation from Chapter 9, problem 14. Take the graph G which consists of a 5-cycle and a single vertex v_6 attached to each vertex of the cycle. Such graphs are sometimes called *wheels*. The 5-cycle inequality is valid for the 5-wheel.

   (1) What is the dimension of the face induced by the 5-cycle inequality? What is the dimension of the stable-set polytope of the 5-wheel?
   (2) Lift this face to a facet by adding a term representing the variable x_6 to the inequality.

7. Chapter 9 problem 16. A premise of this problem is that your I.P. solver will use extensive branch-and-bound to solve this program. In fact in the 12+ years since this problem was written, technology has improved considerably, and many modern solvers will automatically generate knapsack cover-type cutting planes and solve the problem without branching at all. The intent of the original question is for you to try generating some of the initial relevant cover inequalities and see how much they helped the solver. Since finding these inequalities is now standard operating procedure for good solvers, please do the following:

   (1) Use your solver to find a optimal solution to the L.P. relaxation of the given I.P.
   (2) Find 3 obvious cover-style inequalities that cut the initial relaxed optimum.
   (3) Use your solver to find an optimal solution to the I.P. Report whether your solver needed to resort to branch-and-bound, and, if such information is available, what types of cuts and how many it used prior to the first branching.

For items (1) and (3) please include relevant computer printouts.

8. What are the facets of the symmetric travelling salesman polytope for the complete graph on 5 vertices?

Reading:

Chapters 10 and 11.

Tentative schedule of presentations:

Text references are to [3].

Friday, November 26th

(1) T.J. Yusun, 19-19.3, corner relaxations.
(2) Sara Taghipour, [1].
(3) Mehrnoush Malekesmaeili, 18-18.3, modelling combinatorial optimization problems with semidefinite programming.

Wednesday, December 1st.

(1) Farzana Sultana, 16, mixed integer programming computation.
(2) Yong Zhang, [2].

Friday, December 3rd.

(1) Ali Nadaf, 17-17.?, symmetry in integer linear programming.
(2) Piyashat Sripratak, 17-17.?, symmetry in integer linear programming (continued).
(3) Tanmay Deshpande, 15.4, polynomial optimization.
(4) Maria Tamayo, 15.4, polynomial optimization (continued).

Please see me if any information appears to be incorrect.

Each presentation should be about 25 minutes. As part of the grading, students will submit a copy of the overheads used for the talk.

References

[1] Makoto Asano and Hiroshi Ohta. Single machine scheduling using dominance relation to minimize earliness subject to ready and due times. *International Journal of Production Economics*, 44(1-2):35–43, 1996.

[2] Samuel Burer, Renato D. C. Monteiro, and Yin Zhang. Maximum stable set formulations and heuristics based on continuous optimization. *Math. Program.*, 94(1, Ser. A):137–166, 2002.

[3] Michael Jünger, Thomas Liebling, Denis Naddef, George Nemhauser, William Pulleyblank, Gerhard Reinelt, Giovanni Rinaldi, and Laurence Wolsey, editors. *50 years of integer programming 1958–2008*. Springer-Verlag, Berlin, 2010. From the early years to the state-of-the-art, Papers from the 12th Combinatorial Optimization Workshop (AUSSOIS 2008) held in Aussois, January 7–11, 2008.