First Homework Assignment for Math 408 and 708

Due: Friday, September 24th, in class.

All references are to the Wolsey text.

Problems for Math 408 and 708:

1. Chapter 1, problem 1.

2. Chapter 1, problem 2.

3. Take your nine digit student id number and replace each 0 with a 5. Then the digits are $a_1, a_2, \ldots, a_9$. Your personalized knapsack problem is:

$$\text{Maximize } \sum_{i=1}^{9} x_i \quad \text{subject to } \sum_{i=1}^{9} a_i x_i \leq \frac{1}{2} \sum_{i=1}^{9} a_i \quad \text{and } x_i \in \{0, 1\} \text{ for } i = 1, \ldots, 9.$$

Use a software package to solve this integer program. This should be possible using, for example, `Microsoft Excel` or the student demos of `AMPL` and `CPLEX` available at `http: www.ampl.com`. You may also request remote access to the Surrey computational server `Op` where the student demos will be made available.

4. Chapter 1, problem 7.

5. Consider the problem of colouring the vertices of a graph $G = (V, E)$ using the minimum possible number of colours such that no edge connects vertices of the same colour. Formulate this problem as an integer program. You can assume that you have an a priori upper bound K for the number of colours you will use, perhaps $K = |V|$ or something smaller based on knowledge of the graph.

Problems mainly for Math 708:

6. A *magic square* is an arrangement of the numbers $1, 2, \ldots, n^2$ in an $n \times n$ box such that each row, column and diagonal has a constant sum. Formulate as an integer program the problem of finding a magic square maximizing the sum of the entries in its upper left quadrant. You may assume n is even.

7. Chapter 1, problem 10.

8. Consider the problem of allocating storage (memory) dynamically in a computer. Model the memory as a simple array indexed by the positive integers. Suppose we are given a series of n requests to use an array of size s_i from arrival time r_i to departure time d_i. We would like to find the minimum memory size that will accommodate these requests (and a way to do it). Formulate this problem as an integer program.

Reading:

Chapters 1 and 2.