Due: Friday, October 21st (9:30 a.m.)

Reminders

The midterm will take place in class on Monday, October 24th. It will cover material from class up to Friday, October 21st.

Note that Skytrain maintenance is taking place that day, please leave early to make sure you arrive in time for the test.

Reading

From the textbook, reread Section 1.2.2, and then read Section 5.2.4.

Chapter 2 of Applegate, Bixby, Chvátal and Cook, which shows several interesting problems that can be modelled as a TSP, and hence as an integer program.

Problems for Math 408 and Math 708

All problems to be submitted via Canvas. Please submit a single file names `hw3.pdf` containing all your written work, along with files `hw3.dat` and `hw3.mod` for the AMPL question (question 3). Please make sure to write your name on the first page of `hw3.pdf` and in the comments of `hw3.dat` and `hw3.mod`.

1. Textbook Exercise 1.4.

2. Consider solving a maximization problem by branch-and-bound. Suppose that Figure 1 represents part of the branch-and-bound tree. The value inside each node represents the solution to the associated relaxation of the problem; nodes are coloured blue if the relaxed solution is non-integer, green if it is integer, and red if no relaxed solution exists because the problem is infeasible.

   a. Which node(s) still need to be expanded?

   b. Which node(s) are candidate for the optimal solution?

   c. Suppose that before beginning the search, we obtained an integer feasible solution with value 30. Which node(s) would we no longer have to expand?

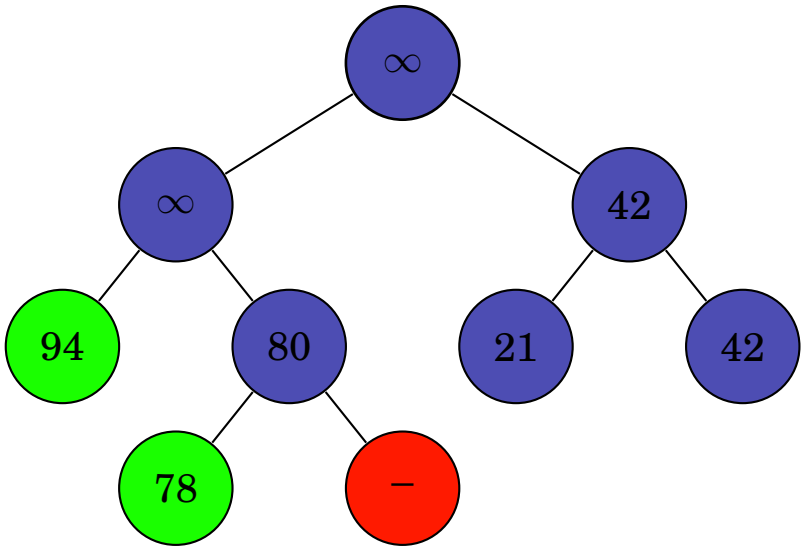

Figure 1: Branch and bound tree for problem 4.

3. Exercise 20.3 (a) and (b) from the AMPL book, available at:

`https://ampl.com/learn/ampl-book/`.

4. Consider the complete graph on 4 vertices. Use the last 6 digits of your student id to weight the 6 edges of the graph.

a. How many Hamiltonian cycles (i.e. cycles connecting all 4 vertices) are in this graph? Give the weight of each cycle.

b. Does your weighting satisfy the triangle inequality?

c. Run the nearest neighbour heuristic on the graph, beginning by traversing the minimum weight edge. What is the approximation ratio that you get from this algorithm?

5. Using the complete graph on 4 vertices, show that if the weights on the edges do not satisfy triangle inequality, then Christofides heuristic can return a solution with approximation ratio 5.

Additional Problems for Math 708

6. Textbook Exercise 4.5.

7. Returning to question 5, what is the worst approximation ratio you personally could get over all possible assignments of weights to edges? What is the worst approximation ratio that any student could get?