Due: Monday, November 26th (in class)

Reminders

Please fill out the on-line course evaluation when available.

The final exam will take place Wednesday, December 12th at noon in SUR 5360.

Reading

Note that column generation and the 1-tree relaxation of the TSP, as well as appearing in Chapter 8 of the textbook, are covered in Applegate, Bixby, Chvátal and Cook, in Sections 11.4.5 and 4.1, respectively. Chapter 12 gives some additional ideas about how to work with very large scale instances, using different relaxations than in class.

Problems for Math 408 and Math 708

1. Show that $F(x) = |x^3| + |x|$ is a convex function, and describe its subgradients.

2. Using Google Maps, or similar navigation software, find pairwise driving distances between the following locations:

- The Bell Centre in Montreal (or Centre Bell, if you prefer).
- The Scotiabank Arena in Toronto (I was surprised to learn it is no longer the Air Canada Centre).
- The TD Garden in Boston.
- Madison Square Garden in New York.
- Little Caesars Arena in Detroit.
- The United Centre in Chicago.

The distances do not have to be very exact, and you can assume that they are symmetric (route lengths in different directions may vary slightly due to one way roads, etc.) by just using one of two the distances, or, better, taking the average of the two. Consider looking for a shortest tour through theses 6 sites.

a. Write down the relaxed Travelling Salesman Problem (TSP) formulation of this problem where the only constraints are those checking that the weight of edges are between 0 and 1, and the total weight of edges on a vertex sum to 2.

b. Does the solution you get also satisfy the subtour elimination constraints? If not, find one subtour constraint that is violated by the relaxed solution.

c. Using the Bell Centre as vertex 1, what is the minimum cost 1-tree on the graph?

3. Textbook exercise 8.4.

4. Show that the greedy heuristic applied to the maximum unweighted matching problem guarantees an approximation ratio of $\frac{1}{2}$, but not $\frac{1}{2} + \epsilon$ for any $\epsilon > 0$.

5. Using a mathematical software package such as MATLAB, generate 15 points in $\mathbb{R}^2$ by choosing each coordinate to be an integer uniformly at random in the range $[1, 100]$. Compute the matrix of pairwise distances between these points. (You can round the distances to the nearest integers.) Please include a computer printout showing the points and the pairwise distances.

Apply the Christofides heuristic to find a good tour through these points. Draw a picture illustrating the points, the minimum spanning tree, the matching edges and the found tour. Can you improve this solutions by exchanging a pair of vertices?

Additional Problems for Math 708

6. Consider the knapsack problem from the previous assignment:

$$\max \mathbf{e}^t \mathbf{x} \text{ s.t. } \quad \mathbf{x} \in \{0,1\}^6, \quad 5x_1 + 3x_2 + 8x_3 + 9x_4 + 13x_5 + 8x_6 \leq 15.$$

Construct the Lagrangian dual with parameter λ by dualizing the knapsack constraint. Describe the Lagrangian dual function $Z(\lambda)$ as a piecewise linear function.

7. Textbook Exercise 8.2.

8. Implement the 2-OPT heuristic for TSP. Test on instances where each node is a random uniformly distributed point on the unit square, with $c_{ij} = distance$. Try with $n = 10$ and $n = 100$.

Please submit your answer to problem 8 directly to the teaching assistant by e-mail (`xla97` at `sfu` dot `ca`). All file names should begin: `math_408_1187_name_hw5_q8` where `name` is your family name. Submit the relevant .dat, .mod and a .pdf file showing your output in a single e-mail. If you would like to submit additional questions via e-mail, you may do so, but only if they are *typeset*. (Do **not** include documents that are produced by a scanner.) If you wish to do this, include your remaining answers either in a single .pdf file named: `math_408_1187_name_hw5_qall`, or with one .pdf file per question names `math_408_1187_name_hw5_q1` (etc., as appropriate). Here again, `name` is substituted with your own family name.

Schedule of graduate presentations

Each graduate student will present a brief introductory lecture on an additional topic in integer programming. This should contain substantial mathematical content and be understandable to the undergraduate students. The talks will be 20 minutes, followed by a 5 minute question period. Overheads will be submitted as part of the grading. The tentative schedule and topics are as follows:

Monday, November 26th **Jasdeep Dhahan**, Fourier Elimination (Section 3.1 in the text).

Wednesday, November 28th (early) **Chloe Li**, The Circulation Cone (from Section 4.3.1 in the text).

Wednesday, November 28th (late) **Navpreet Kaur**, Weekly Scheduling Modules for Traveling Therapists (Socio-Economic Planning Sciences 47 (3) 2013, pp. 191-204).

Friday, November 30th (early) **Haggai Liu**, Solving Linear Diophantine Equations (from Section 1.5.2 in the text).

Friday, November 30th (late) **Shawn Yan**, Graver Bases (Sections 3.1 and 3.2 in De Loera, Hemmecke and Köppe).

The criteria for evaluating the presentation will include:

- The presentation highlights critical aspects of the report, and is suitable for the audience.
- Ideas are presented clearly and logically.
- Live presentation is well prepared, accurate, and professionally delivered. Questions are answered appropriately.
- Overheads are clear, well-formatted, and have few errors.