Quiz: Friday, February 11th (in class)

Reminder

The midterm will take place in class on **Friday, February 4th**.

Reading

For Wednesday, Section 9.2.
For Monday, February 7th, Section 9.3.
For Wednesday, February 9th, Section 10.1.
For Friday, February 11th, Section 10.2.

Assignment questions

Section 9.1: 1, 2.
Section 9.2: 1, 2, 3.
Instructor questions:

1. Let $a_0, a_1, a_2, \ldots$ be an infinite sequence of non-negative integers and $A(x) = \sum_{n\geq 0} a_n x^n$ the corresponding generating function. Express in terms of $A(x)$ the generating function $B(x)$ of each of the following sequences of non-negative integers. For each part, justify your answer. Note that the formula for $B(x)$ should not contain a summation symbol or an infinite sum; it has to be a closed form involving a finite number of terms, although your justification can use summations.
   (a) $0, 0, 0, a_3, a_4, a_5, \ldots$.
   (b) $0, 0, 0, 0, a_0, a_1, a_2, \ldots$.
   (c) $a_0, 0, a_2, 0, a_4, 0, a_6, \ldots$.
   (d) $a_0, 2a_1, 4a_2, 8a_3, 16a_4, \ldots$ (i.e. the sequence $(2^n a_n)_{n\geq 0}$).
   (e) $a_0, a_0 + a_1, a_0 + a_1 + a_2, a_0 + a_1 + a_2 + a_3, \ldots$ (i.e. the sequence $(\sum_{i=0}^{n} a_n)_{n\geq 0}$).
2. Suppose that $A(x) = \sum_{n=0}^{\infty} a_n x^n$, $B(x) = \sum_{n=0}^{\infty} b_n x^n$, and that $A(x)\frac{1}{1-x} = B(x)$. Give an expression for b_n, in terms of $a_0, a_1, \ldots, a_n$. Justify your answer.
3. We now investigate the generating function of Fibonacci numbers F_n, defined by $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ if $n \geq 2$. The first Fibonacci numbers are $0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots$. Based on the definition of F_n, prove that the generating function of these numbers
$$F(x) = \sum_{n\geq 0} F_n x^n$$
satisfies the following identity:
$$F(x) = \frac{x}{1-x-x^2}.$$

Some other questions worth trying

Note that there are supplementary exercises at the end of each chapter, these may in some cases make good review questions for the midterm.