AN OPEN-ACCESS CLICKER QUESTION BANK FOR NUMERICAL ANALYSIS[*]

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Abstract. We present a question bank consisting of over 250 multiple-choice and true-false questions covering a broad range of material typically taught in an introductory undergraduate course in numerical analysis or scientific computing. The questions are ideal for polling students during lectures by means of a student response system that uses *clicker* remotes or smartphones running a suitable app. We describe our experiences implementing these clicker questions in a recent class and provide evidence of their effectiveness in terms of testing students' prior knowledge, gauging understanding of new material, increasing participation, and especially improving student satisfaction. Our conclusions are supported by a mid-semester student survey as well as anecdotal observations. The question bank has been released as an open-access educational resource under a Creative Commons license (BY-NC-SA) for free use by the mathematics community.

Key words. numerical analysis, clicker polling, student response systems, educational technology, Matlab

AMS subject classifications. 65-01, 97Nxx

1. Introduction. Student response systems, commonly referred to as *clickers*, have become commonplace in university classrooms as a way to interactively probe students' understanding of course material and to engage them more actively in lectures. These systems allow polling of students through a combination of specialized hardware and/or software that record, aggregate and display student responses in real-time. Clickers serve a variety of useful pedagogical purposes such as: testing prior knowledge, gauging student understanding of new concepts, providing instantaneous and visual feedback to students, increasing classroom participation, etc. A major advantage of being able to poll the entire class in this manner is that one can immediately identify situations where a significant number of students are confused or lacking understanding on an important point, thereby empowering the instructor to adjust their lecture on-the-spot to address the problem.

Although clickers have been utilized extensively and effectively in many university classes, it has been predominantly in lower division courses; and in the case of mathematics, attention has been predominantly on development of clicker questions for introductory courses in calculus and linear algebra. There is a particularly conspicuous gap in openly-available clicker resources that cover material in the area of numerical analysis, which is identified next.

1.1. Review of Clicker Resources: The Numerical Analysis Gap. The use of short multiple-choice or true-false questions in a lecture setting was first proposed for physics courses by Eric Mazur in the form of *ConcepTests* [7, 12]. This approach was rapidly incorporated in other disciplines, and the polling of students during lectures was made easier and more interactive with the development of electronic student response systems. These clicker devices, along with accompanying software, allow students' responses to be compiled immediately, presented as a sum-

mary table or bar chart, and then discussed further in class. An excellent literature review of clickers and their pedagogical uses in a wide range of disciplines is provided by Caldwell [5]. Clicker questions have become pervasive in mathematics instruction and there are numerous studies of clickers in entry-level undergraduate mathematics courses [3, 11, 13, 14, 17, 18]. These studies have established that use of clicker polling in lectures is "yet another source of a valuable learning tool for students" [13] and can generate a number of positive outcomes for both instructors and students. The most obvious and prosaic benefit for instructors is in terms of increased attendance, although they also clearly benefit from gaining an immediate measure of students' current understanding of course concepts. From the student's point of view, the advantages include immediate and regular feedback on their learning, an ability to gauge their own knowledge against that of their peers, and improved satisfaction with their learning experience. Regular participation in clicker polls can also yield the interesting spin-off effect of stimulating other forms of in-class participation such as peer discussions or think-pair-share, thereby engaging students even more actively in lectures.

Devising interesting and insightful clicker questions can pose significant challenges. A concise and practical resource that provides useful tips on how best to formulate questions and implement them effectively in lectures is the *"Clicker Resource Guide"* by Wieman et al. [22]. In this guide, the authors identify ten question types that serve various purposes in a classroom setting, such as:

1. testing comprehension of assigned readings,
2. assessing recall of lecture points,
3. performing a step in a calculation,
4. surveying student opinions,
5. revealing pre-existing knowledge or thinking,
6. testing conceptual understanding,
7. applying ideas in a new context,
8. predicting results of a demo or simulation,
9. drawing on knowledge from everyday life,
10. relating different representations (mathematical, graphical, algorithmic, etc.).

Devising questions that satisfy such a wide variety of purposes for all topics in a course can be a daunting task, but thankfully there already exist extensive collections of openly available questions for certain subjects. The two most notable resources in mathematics are the GoodQuestions project for calculus [21], and the MathQUEST/MathVote [6, 23] project which covers a broader range of the first- and second-year undergraduate curriculum in calculus, linear algebra, differential equations and statistics. Several textbooks come bundled with a clicker resource [1, 9, 16] but these questions are subject to copyright and so are not freely available.

When it comes to upper-year undergraduate courses like numerical analysis, there are comparatively few open collections of clicker-type questions. Some multiple-choice and true-false questions can be found among the exercises in certain numerical analysis textbooks, the most noteworthy being Heath's text [8]. The only freely-available question bank we are aware of is distributed along with an open textbook by Kaw et al. [10], but both the number of questions and diversity of topics are somewhat limited.

1.2. Aims and Outline. The main purpose of our paper is to remedy this "numerical analysis gap" and our aims in this respect are three-fold. First of all, we provide an account in sections 2 and 3 of our efforts to construct a question bank that

covers much of the standard curriculum in a third-year undergraduate course at Simon Fraser University called *Numerical Analysis I* (MACM 316 for short). Questions span the theory and application of algorithms for scientific computing and are posed as either true-false or multiple-choice questions (with 3 to 5 choices). Our intent is to come up with a sufficiently large collection of questions (at least 250 in number) that not only provides comprehensive coverage of the course material but also permits the instructor some flexibility in terms of both level of difficulty and question type. Because algorithms and Matlab programming are an integral part of MACM 316, we also include questions that test students' understanding of basic algorithms in the form of Matlab code, such asking them to identify bugs in erroneous code or to interpret tabulated or graphical output.

Secondly, we are releasing the clicker question bank as an open educational resource (OER) that should be of significant value to other teachers of numerical analysis from the applied mathematics community. We describe in section 3 how the questions and solutions are conveniently written in LaTeX, and macro files and slide templates are provided so that questions can be easily incorporated into other instructors' lectures. Section 4 is a brief "how-to guide" that explains how we have integrated the clicker questions into our own lectures, along with some helpful in-class tips. By releasing this resource under a Creative Commons Share-Alike (CC BY-NC-SA) license, we are strongly encouraging readers to contribute to this OER themselves by making corrections or improvements to existing questions, and more importantly by proposing new questions of their own that expand on the topics already covered. The following web site provides the complete resource in both `zip` and `tar.gz` formats for easy download:

http://www.sfu.ca/~jstockie/NAclicker

Our third and final aim is to share our experiences implementing the clicker question bank in a numerical analysis class during a recent semester (January to April 2020) and to illustrate how it has led to significant improvements in student satisfaction as well as the classroom learning environment. Support for these claims is provided in section 5, which summarizes the results of a mid-semester student survey and also provides anecdotal observations comparing our experience teaching with clickers to several previous offerings of the same course without clickers.

If you have ever been tempted to experiment with clickers in your own numerical analysis course but were intimidated by the amount of work involved in writing good questions, then this just might be the OER for you!

2. Course Overview. We begin with a description of SFU's numerical analysis course MACM 316 to set the context for the development of the clicker question bank. MACM 316 is a course called *Numerical Analysis I* that covers the elementary topics from numerical analysis listed in the syllabus provided in Table 1. These are fairly standard topics in many introductory numerical analysis courses taught in universities around the world, and the topics map easily to the contents of most textbooks, one example being the seminal *"Numerical Analysis"* by Burden, Faires and Burden [4] (which is the required text for MACM 316). The ordering of topics in the course syllabus is slightly different from that in many books including [4] and is solely a personal choice of the instructor (JMS). A teaching semester at SFU consists of roughly 40 hours of instruction spread over 13 weeks and courses are parceled into 50-minute lectures taught three times per week.

2.1. Student Demographics. MACM 316 is a core course in the *Mathematics and Computing Science* (MACM) program at Simon Fraser University. Roughly two-

TABLE 1

Syllabus for MACM 316. The numbering of topics is the same as that used in the question bank, and the corresponding section number is listed from Burden, Faires and Burden [4].

#	Topic	BFB Section
1.	Introduction:	
1a.	Floating-point arithmetic and errors	1.2
1b.	Review of concepts from calculus	1.1
2.	Nonlinear equations:	
2a.	Bisection method	2.1
2b.	Fixed point method	2.2
2c.	Secant and Newton's methods	2.3
2d.	Convergence of iterative algorithms	1.3
3.	Systems of linear equations:	
3a.	Review of linear algebra	6.1, 7.1
3b.	Gaussian elimination, pivoting and matrix factorization	6.1, 6.2, 6.5
3c.	Plotting power laws (interlude)	
3d.	Iterative methods for sparse systems	7.3, 7.4
4.	Function approximation:	
4a.	Polynomial (smooth) interpolation	3.1, 3.3
4b.	Spline (piecewise) interpolation	3.5
4c.	Least squares fitting	8.1, 8.2
5.	Differentiation and integration:	
5a.	Numerical differentiation	4.1, 4.2
5b.	Numerical integration or quadrature	4.3, 4.4, 4.6
6.	Ordinary differential equations (ODEs):	
6a.	Background on ODE initial value problems	5.1
6b.	Euler's method	5.2
6c.	Taylor and Runge-Kutta methods	5.3, 5.4
6d.	Systems of first-order ODEs	5.9

thirds of students are enrolled in the School of Computing Science, where the course is a graduation requirement. The remaining students are divided almost equally between programs in the Faculties of Science and Applied Science (Engineering). The prerequisites for MACM 316 are two first-year classes in differential and integral calculus, plus an introductory linear algebra class. Some computing experience is required (or at least strongly recommended) because algorithms are central and students are obliged to submit Matlab code as part of their assignment solutions. Major theorems and results on error estimates and convergence are all covered, but the emphasis is more on practical implementations of algorithms than on the theory behind them.

The usual enrollment in MACM 316 hovers around 200 students, with roughly 25% in year 2 and the remainder in years 3-4 of their undergraduate programs. The main point of this remark is that the class is a relatively large one, and it is also an upper-year course so that most students are already "more experienced". This distinguishes the student cohort in MACM 316 from that in most other clicker-based courses because clickers are most often implemented in large introductory classes [2, 3].

2.2. Grading Scheme. The serious computing component of this class is reflected in the grading scheme in Table 2 which puts significant weight on computing assignments. We also highlight the 3% of the final grade that is assigned to participa-

TABLE 2
Grading scheme for MACM 316.

Quizzes (weekly, 10 mins long, in class)	18%
Computing assignments (bi-weekly)	24%
Clicker questions (participation only)	3%
Midterm test	25%
Final exam	40%

tion in the in-class clicker polls. These 3 points are an "all-or-nothing" grade that a student receives only if they respond to at least 75% of clicker questions posed during semester. Our aim is to encourage students to be present in class and to participate in the clicker polls, but to otherwise avoid imposing undue pressure on students to obtain the correct answer.

3. Structure of the Clicker Question Bank. The clicker question bank consists of a mix of both multiple-choice and true-false questions based on the material from the syllabus in Table 1, which lists topics that are easily mapped onto chapters/sections from most numerical analysis textbooks. In authoring the questions, we aimed to employ a variety of the question types identified in the introduction (numbered 1–10) although we concede that this initial release of the question bank includes very few of types 1, 4 or 9.

Approximately 80% of questions were authored by ourselves, with the remaining 20% derived from a few specific sources. The bulk of these latter questions were adapted from the numerical analysis texts by Heath [8] and Kaw et al. [10]. A smaller number of questions on background material from calculus and linear algebra were based on material from the GoodQuestions [21] and MathQUEST/MathVote projects [23].

The clicker question bank (consisting of LaTeX source, image files and Matlab code) can be obtained from the URL provided at the end of the introduction. Below are a few main features of the question bank that will help readers who are interested in implementing the questions in their own courses and/or adding questions of their own:

- In total there are 267 questions, of which 242 are multiple-choice and the remainder are true-false. The multiple-choice questions allow for 3, 4 or 5 choices (labelled A,B,C,D,E).
- Questions are encoded in LaTeX to ensure high-quality typesetting of the mathematics and to allow them to be easily modified and cut-and-paste into other LaTeX documents. Several LaTeX files deserve explanation:
  (a) `clickerQbank.{pdf,tex}`: A document containing all clicker questions that generates a "compact" 90-page listing of questions and solutions. Each question is defined as an item within a list environment called `clicklist`, and there is a macro named `\qitem...` defined for each question type. For example, a multiple-choice question with four choices (A,B,C,D) is encoded using the macro `\qitemMCfour` and is passed 8 arguments as follows:

```
\begin{clicklist}
 ...
\qitemMCfour{You have a system of three linear equations
```

```
    with three unknowns. If you perform Gaussian
    elimination and obtain the reduced row echelon form
    \begin{gather*}
      \left[ \begin{array}{rrr|r}
          1 & -2 & 4 &  6 \\
          0 &  1 & 0 & -3 \\
          0 &  0 & 0 &  0
      \end{array} \right]
    \end{gather*}
    then the system has \dots}% ARG1: question statement
  {no solution}                % ARG2: response (A)
  {a unique solution}          % ARG3: response (B)
  {more than one solution}     % ARG4: response (C)
  {infinitely many solutions} % ARG5: response (D)
  {4}          % ARG6: correct answer (1=A, 2=B, 3=C,...)
  {The last equation reads ''$0=0$'' so $x_3$ can be any
   real number. Strictly (C) is also correct, but (D) is
   the most accurate answer.} % ARG7: solution details
  {MAH}                        % ARG8: author or source
  ...
\end{clicklist}
```

The formatted output for this question (numbered Q3b-16 in the question bank) is displayed in Figure 1. Additional macros are defined for the `clicklist` items `\qitemMCthree`, `\qitemMCfive` and `\qitemTF` corresponding to the other question types.

(b) `clickerbeamer.{pdf,tex}`: A beamer [20] implementation of the `\qitem...` macros that instead generates a pair of slides – one containing the question by itself, followed by a second showing both question and answer. An example is pictured in Figure 2.

(c) `clickerslides.{tex,pdf}`: A simple non-beamer slide implementation.

(d) `clickerdefs.tex`: Defines several list counters and LATEX macros that are common to questions in all formats.

- Some questions include images, plots, tables, Matlab code, or numerically-computed output. All images are provided in both `.eps` and `.pdf` formats within the subdirectory `figures/`.

- All plots are generated using Matlab and the corresponding code can be found in the subdirectory `figures/matlab/`. This permits any instructor to easily modify a question by changing parameters, plot format, etc.

The question depicted in Figures 1 and 2 has the interesting feature that there are two valid responses (C,D). When this question was actually presented in class, students were equally-split between responses (C) and (D), which then naturally initiated a follow-up discussion around why there were multiple correct answers and why (D) was the "best" response. This is just one example of when an instructor must be prepared to respond to poll results by spontaneously allowing students to discuss and work through the solution for themselves rather than just uncovering the answer slide and moving on. Most questions in the question bank do have one unique answer, but there are several with multiple valid responses that can present opportunities for lively

FIG. 1. *Formatted output from* `clickerQbank.pdf` *for a sample multiple-choice clicker question having 4 choices, typeset using the macro* `\qitemMCfour`.

(a) First slide with question only.

(b) Second slide showing question and answer.

FIG. 2. *Beamer slides from* `clickerbeamer.pdf` *for the same multiple-choice question in* Figure 1.

classroom debates of this sort. Indeed, Wieman et al. [22] remark that "such questions can generate the most educationally productive discussions." Moreover, clicker polls and the animated peer discussions that precede them provide a mechanism for what mathematics educators identify as "embodied actions to build up dynamic embodied concepts in mathematics" [19].

Another noteworthy feature of the question bank is that there are several groups of closely-related questions, consisting of minor variations on a common theme or differing levels of difficulty. On the one hand, this provides instructors with some variety and flexibility to focus on specific aspects of the material. But while it is not intended that all such related questions would necessarily be posed together in a single lecture, mathematical cognition researchers such as Tall [19] have recognized that such repetitive questioning encourages "recognition of similarities, differences and patterns; repetition of actions to make them routine."

This question bank should be considered a "work in progress" and there remain several gaps in coverage corresponding to topics that are part of the standard introductory numerical analysis curriculum. Three examples of such gaps are Newton's method for nonlinear systems of equations, matrix eigenvalue problems, and numerical stability and implicit methods for ODEs. We would also like to include more questions that illustrate concrete applications of algorithms to problems from physics, engineer-

FIG. 3. *(Left) Two versions of the iClicker radio-frequency remote. (Middle) iClicker RF base station. (Right) Laptop and smartphone running the* iClicker REEF *app through a Wi-Fi connection. Source:* http://www.iclicker.com.

ing, biology, economics or other fields, all of which we hope to include a future release of the question bank.

4. Mechanics of In-Class Clicker Polling. For this section only, we switch to using the personal pronoun "I" since one of us (JMS) was the instructor for the MACM 316 class. The aim here is to provide a "how-to guide" of sorts that permits the interested instructor to incorporate clicker polling into their own lectures as quickly and easily as possible. Some of what follows relates specifically to the iClicker system (http://www.iclicker.com) which I selected because it integrates so seamlessly with the Canvas learning management system (http://www.infrastructure.com/canvas) used at SFU. This requires that all students purchase or borrow an iClicker radio-frequency remote, two versions of which are pictured in Figure 3(left). Student responses are gathered by means of a base station that is connected via USB to a computer running the *iClickerClassic* app. iClicker has an alternate web-based system in which students use smartphones and laptops running the *iClicker REEF* app instead, responses are transmitted through a Wi-Fi connection, and the base station is eliminated in favour of an internet-connected instructor app called *iClicker Cloud* (shown in Figure 3(right), which also integrates with Canvas). During this initial trial of the question bank, I decided to use the remote-only *iClickerClassic* option so as to minimize any difficulties managing input from multiple devices, as well as escaping problems with intermittency in classroom Wi-Fi coverage. However, in future offerings of MACM 316 I would envisage switching to *iClicker Cloud*, not least because the most common complaint on the student survey (see section 5) was the cost of the remote device, and students expressed a strong preference for using smartphones to submit their responses. Indeed, it is likely that the next MACM 316 section I teach will be offered remotely due to the Covid-19 pandemic.

iClicker is only one company that markets student response systems, and several other systems are available. For example, Turning Technologies manufacture their own remote devices with a corresponding app that is also supported by Canvas. Other learning management systems such as TopHat (http://www.tophat.com) and Acadly (http://www.acadly.com) have their own built-in polling systems that don't require specialized hardware but permit students to use their smartphones or laptops.

I found that it took a few lectures to familiarize myself with the process of clicker polling, to get used to the software, and to develop my own routine for incorporating clicker questions into lectures. The iClicker system makes this process relatively easy and painless, and permits significant flexibility for customizations that suit individual instructors. I describe below the general procedure I followed when polling students,
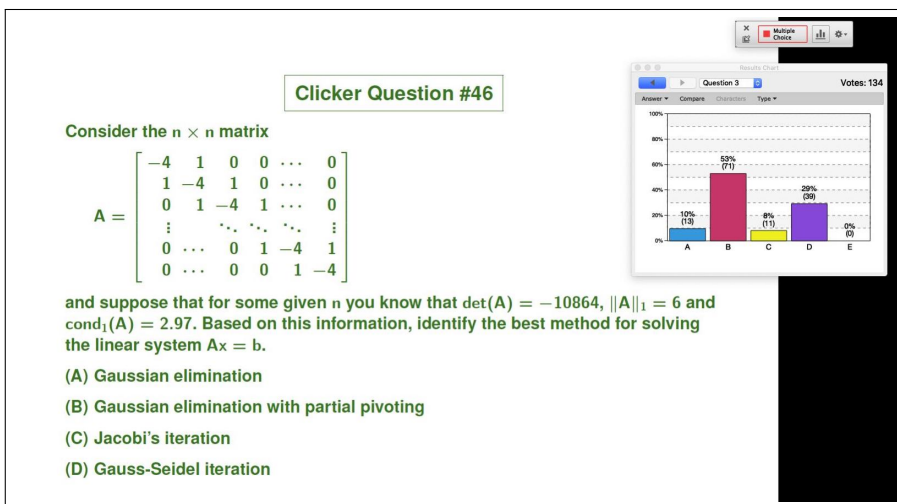
FIG. 4. *Lecture slide displaying a clicker question, overlaid by the* iClicker Classic *widget (top right) and a bar plot summarizing student responses (middle right).*

along with a few tips that other instructors may find helpful:

- Polling is particularly easy to implement in a class where lecture notes are displayed using an LCD projector connected to a laptop that is running the iClicker app. In MACM 316, I distribute "skeleton" lecture notes, typeset in LaTeX, that provide basic definitions, theorems, background material and examples, and leave students to fill in the blanks during class. Clicker questions are easy to cut and paste from the LaTeX question bank into my presentation slides, though I do not include them in the student skeleton notes distributed before lectures. The clicker questions can also be used in a class taught on the whiteboard or blackboard, but one has to be prepared to toggle back-and-forth from the LCD projector when displaying questions and poll results.
- With the *iClickerClassic* app running in the background, a small widget is displayed on top of the slide window that contains a play/stop button and a plot button (see Figure 4). Whenever the notes are advanced to a slide containing a clicker question, I launch the poll from the iClicker widget which then starts a timer and displays a live counter of the number of responses. I typically provide students between 45–60 seconds to select a response and I also give a 10-second warning before closing each poll. It is possible to configure the app to use a count-down timer instead so that students can see precisely how much time is left. On completing a poll, clicking the plot button on the widget displays a histogram of student responses (see Figure 4).
- I normally pose between 3 to 5 clicker questions during each 50-minute lecture, with the number depending on the material being taught. The questions are grouped in clusters of 2-3 to minimize the overhead required when asking students to get their clickers ready, changing between applications, and "switching gears" in my delivery. Working through each question cluster typically requires at least 5 minutes, which is not only a welcome break for students but also helps me avoid my own natural tendency to speed up and "steam-roll" through the lecture material.
- I imposed no set pattern on the timing of questions in any given lecture,

variably using them as initial warm-up problems, or to check in on student understanding after teaching a new concept, or as a final wrap-up, or simply to introduce a break. The key is to be flexible.

- I encourage students to discuss questions with their neighbours and I occasionally walk around the room to listen in on conversations and gauge what difficulties students might be having. Quieting students down after a particularly vigorous discussion may sometimes take a few extra moments, but I believe that this is time well spent and not at all disruptive.
- Upon revealing the histogram of poll results, if the majority of students chose the correct answer (at least 75%) then I normally advance directly to the solution slide, highlight an important point or two, and move on. Otherwise, time permitting, I devote some extra time to a follow-up discussion or work through some solution details to ensure that students who answered incorrectly understand the source of their error.
- If a question receives an unusually high proportion of incorrect responses then I make a special note of it, recognizing that such questions are typically either:
    - overly difficult, ill-posed or confusingly worded: such questions were either be modified or removed from the question bank.
    - challenging or "tricky": which are often good topics for extra discussion. I flag these equations as potential test material and/or devise supplementary examples that explore deeper aspects of the material for future classes.

It is important to recognize that any extra class time spent on clicker polling and discussion has a significant effect on the material that can be covered. Wieman et al. [22] observe that "most instructors do end up deciding to cover slightly ($\sim$10%) less material in their courses after they have started using clickers effectively," and my own experience is consistent with this. However, this is a small price to pay considering the substantial benefits in terms of student satisfaction and improved classroom learning environment.

4.1. Unexpected Switch to On-line Instruction. The Jan-Apr 2020 semester on which this paper is based was an exceedingly unusual one because it coincided with the outbreak of the Covid-19 pandemic. Classes at SFU ran as usual until mid-March (essentially mid-semester) at which time the university decided to shift all classes to an on-line mode of delivery to comply with provincial physical distancing guidelines. Because iClicker remotes function only in a classroom setting with an RF base station, I decided at that point to stop collecting clicker response data and to determine students' participation grades based only on their responses from the first half-semester of clicker questions.

Like many other universities, SFU has responded to the Covid-19 pandemic by moving almost exclusively to an on-line mode of teaching for the next academic year. This means that clicker remotes cannot be used since they operate by communicating radio-frequency signals to a base station. Therefore, at least in the near future, clicker polling in MACM 316 will have to be done via Wi-Fi connection using a system like *iClicker Cloud/REEF*.

Although the mid-semester interruption of this clicker experiment was very disheartening, it did uncover another convenient but unforeseen application of the clicker question bank. Upon the switch to on-line instruction, I simultaneously transformed the weekly in-person quizzes into an on-line multiple-choice format for which it seemed

natural to re-purpose clicker questions. Based on this experience, I can confirm that the clicker bank is also an excellent source of multiple-choice and true-false questions for use on quizzes or tests.

5. Student Satisfaction Survey Results. A student satisfaction survey was held at the mid-point of semester, which coincidentally was only two days before SFU switched all classes to an on-line mode of instruction in response to the Covid-19 pandemic. It was unfortunate that the in-class clicker polling had to be terminated at that point owing to the reliance on iClicker remotes. However, because the survey was completed before the switch, we are confident that the results are unaffected by any of the disruptions that ensued.

A summary of all survey questions and response data is given in Appendix A. The survey consisted of 7 multiple-choice questions (Q1–Q7), another set of 10 statements rated on a Likert scale (Q8), and one free-response comment box (Q9). Several questions were modelled after similar surveys of clicker-based calculus classes [3, 11, 13], and the Likert-scale question Q8 was inspired by the human development class in [2]. A total of 137 responses were received out of 199 registered students. The corresponding response rate of 69% is relatively high for this sort of survey and so we are confident that the results are representative of the entire class. The survey was administered as a Canvas Quiz, which allowed responses to be submitted anonymously and the data easily downloaded.

Rather than describing all survey responses in detail, we will highlight what we view as the most important results and refer the interested reader to Appendix A for more specifics.

- Questions Q2 and Q3 suggest that more than 80% of students thought the clicker questions helped them to improve their understanding of the course material.
- Questions Q4 through Q7 indicate that the majority of students were satisfied with the level of difficulty of the problems, the method of classroom delivery, and the grading scheme based on participation only. Some instructors prefer to assign a clicker grade that gives credit for a mix of participation and correctness [22] and this is easy to set up within the iClicker system. But our experience survey suggests that students derive sufficient benefit without pressuring them to submit correct responses.
- Question Q8 is an especially strong indicator of the high level of satisfaction and positive impacts on students' learning experience. The first three statements (A,B,C) all received more than 80% positive responses (either "agree" or "strongly agree") on the Likert scale, which reflects two important aspects of clicker polling that are also recognized in other studies:
  - Students have a strong preference for the anonymity of a clicker poll over answering questions individually, most likely due to shyness or fear of embarrassment in a large class.
  - Clickers provide a positive break from the rest of lecture.
  Of the remaining statements (D to J) all but H received at least 50% positive responses and at most 20% negative, which reveals the beneficial contributions of clicker polling to the classroom learning environment.
- One result from question Q8 that deserves special mention concerns statement H ("I feel more connected to my classmates when we use iClickers") which had a more mixed response. This exposes a distinct opportunity for us to involve students more actively and collaboratively with each other by

incorporating a *peer instruction* component into the clicker polling process (sometimes also referred to as "think–pair–share"). In this approach, students are provided extra time after the initial clicker poll (but before the solution is displayed) to discuss the poll results with each other and then a second poll is taken to determine any changes in student opinion. Peer instruction is a well-established teaching practice [7, 11, 13, 14, 15, 22] that has proven effective in engaging students more actively in the learning process, improving their critical thinking and communication skills, and building a more collegial intellectual atmosphere. This is something that obviously consumes additional class time, but can be introduced gradually and is not necessary to do for every clicker question.

5.1. Free-form Comments. The final survey question asked students to provide free-form comments in response to the prompt: "Do you have any other comments or suggestions related to the iClicker questions?". The feedback we received was without doubt the most encouraging indicator of student satisfaction with the clicker experiment.

First of all, let us summarize the critical (but mostly constructive) comments we received which can be distilled down to essentially four points:

- *iClicker remotes are too expensive:* The cost of roughly $50 for a remote is a significant expense, although not unreasonable when one considers that nearly 80% of the MACM 316 students needed them in at least one other class (Q1). It is not surprising that students would prefer to use their smartphones, but while the *iClicker REEF* app can be downloaded freely it does require an annual subscription fee of $25. One possible concern about encouraging students to use their smartphones in class is that they may be more easily distracted by alerts from social media and other extraneous apps on their devices.

- *Can you post clicker questions and answers after class?:* In our opinion, the clicker component of this course is a major benefit of attending lectures in person. We have also found that certain clicker questions make excellent test or exam questions, which is why we prefered to not make the questions available to students. Wieman et al. [22] recognize that there are varying opinions about whether or not to provide clicker questions and solutions after class, and conclude that there is "no data that indicates one way or another."

- *Polling time was insufficient, or the question text was too long:* This is a valid complaint and we have already made attempts to shorten the text in some of the wordier questions. We also plan in future to devise more short questions, in particular those of the true-false type.

- *Questions were spread unevenly between lectures:* Although our original plan was to evenly distribute 3–5 questions in every lecture, we failed to account sufficiently for the unpredictable delays involved with conducting polls and discussing the results. Based on our experience, we plan to modify the lecture notes and adjust the distribution of questions to address this concern.

Outside of these few critical comments, we were gratified by the many more remarks of a positive nature that highlighted students' high degree of satisfaction with their learning experience. We have listed a few selected comments below, which are far from an exhaustive list but hopefully convey some sense of the range of potential benefits that clicker polling can have on the learning environment:

- "I think iClickers serve as a useful addition to a class setting. It encourages

active participation and encourages students to engage with the material. In this sense, it seems to work well, especially in a large class setting."

- "It helps to keep my attention in class when we're switching back and forth (helps with short attention span)."
- "I do not feel the pressure of getting a wrong answer in front of a 150 person crowd. I also have more time to think about an answer rather than a sharp student immediately answering so I get more out of the question."
- "At the beginning of the semester I was slightly annoyed with having to buy an iClicker. It was used quite effectively though, and I feel that it was worth the cost overall. It gave me a little extra push to make it to class, without the panic that I get from graded quizzes. I definitely prefer it to the alternatives like TopHat or Canvas quizzes, which are difficult to connect to in some classrooms."
- "I usually enjoy classes with iClickers more because I usually dislike raising my hand so iClickers grant a anonymity that I enjoy."
- "The depth and quality of questions asked made up for the cost and then some. It also helped knowing others in class via discussions, and the anonymity was helpful to know sometimes I am not the only one confused."
- "The participation is a very positive way of doing the questions, rather than pressuring students to get them right. Learning is encouraged, rather than only focusing about getting the correct answer."
- "I love your usage of iClickers, it acts like a little break in between class that helps us keep focused more before/after we do them. They also let us learn at our own pace, getting an answer wrong means I need to work on that certain material again."
- "The questions gave me immediate feedback on how I was doing in the class and what I need more practice on."
- "I particularly like tricky questions, edge case questions, or questions with an unintuitive answer. I feel like these questions really help to solidify my understanding of the material."

5.2. Anecdotal Observations. We make no claim that clicker polling has had any direct connection with improvements in students' grades or other quantifiable learning outcomes. Some previous studies have attempted to do so [7, 11, 14, 15, 17] and although the results suggest there are some positive correlations with student performance, this effect tends to be relatively small. Our focus in this paper is rather on the significant impact that devoting class time to clickers can have on the classroom atmosphere and students' learning experience. Outside of the student survey described above, we have observations from the instructor's perspective of a "before and after" nature (comparing the most recent semester with two other recent offerings of the same course) that provide anecdotal support for the positive effects of clickers in the classroom:

- *Increased attendance:* The percentage of enrolled students attending lectures (when there was no in-class quiz) was between 70–85% and normally lay closer to the high end of this range. This is in comparison with a 50% attendance estimated from other recent offerings of the course. It should be no surprise whatsoever that assigning a portion of the grade that requires attendance in class should increase attendance. Rather, we want to highlight the virtual absence of any negative feedback from the survey regarding the clicker participation grade, nor was there any suggestion that this was viewed as some

form of "coercion".

- *Student questions before/after lectures:* Compared with previous semesters, there was a marked increase in the number of students asking questions during class time (outside that devoted to clicker polling). But what was particularly striking was the long line-up of students waiting to ask questions after class, which often required an additional 20-30 minutes. It is unusual to see such enthusiasm from a large number of students, and we can only hypothesize that perhaps the clicker polling gave students a sense that their instructor is more approachable and cares about their learning.
- *On-line discussions:* A discussion board has always been provided as part of the Canvas container for this class, and there was more than a 100% increase in student postings *before* the move to on-line instruction at mid-semester.

When combined with the survey results, these observations suggest that engaging students more actively in the classroom through the use of clicker polling can have certain beneficial side-effects in terms of not only improving the students' sense of engagement but also enhancing the atmosphere of collegiality both inside and outside of the classroom.

6. Closing remarks. The use of clicker polling in university mathematics classes is commonplace and while many clicker question resources are available for introductory courses, there is next to nothing available for more advanced courses such as numerical analysis. We describe an open-access clicker question bank that we have developed to fill this gap, and which covers a significant portion of the introductory numerical analysis curriculum. Based on the first semester in which the questions were implemented in class, results from a student survey suggest that clickers had a significant positive impact on the student learning experience.

We would like to close with a quote taken from Wieman et al. [22]:

> *"We have found the effective use of clicker questions and discussion can have a transformative impact on both teachers and students, particularly in large classes. Students end up being far more actively engaged in the material and they learn more, and both students and instructors find the course much more rewarding as a result."*

This has certainly been our experience and we strongly encourage readers to consider using clicker polling in their own classroom. Furthermore, with so many universities around the world recently transitioning to remote class delivery in response to the Covid-19 pandemic, clickers present a special opportunity for instructors to engage their students actively in an on-line lecture setting. It is our sincere hope that the open-access clicker question bank described here will help to reduce the "cost of entry" for instructors of numerical analysis who have an interest in experimenting with student response systems.
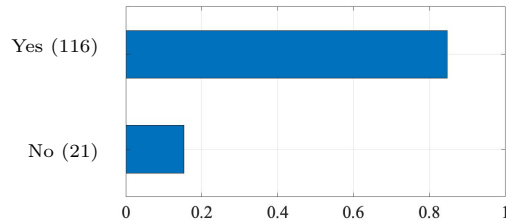
Appendix A. Student Survey Data.

For the first 7 questions (Q1–Q7) the responses are depicted as a bar chart showing the percentage of all 137 completed surveys, and the number of students who selected each response is also shown in parentheses.
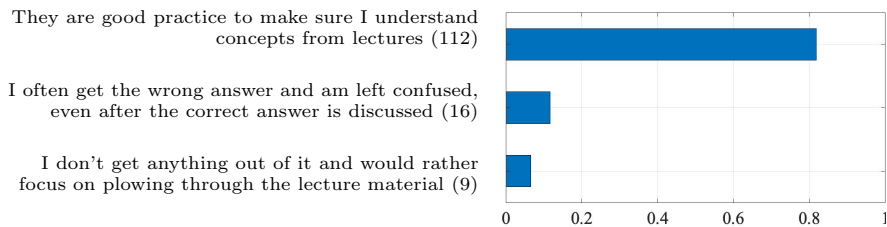
Q1. In how many other classes have you used iClickers before this semester?
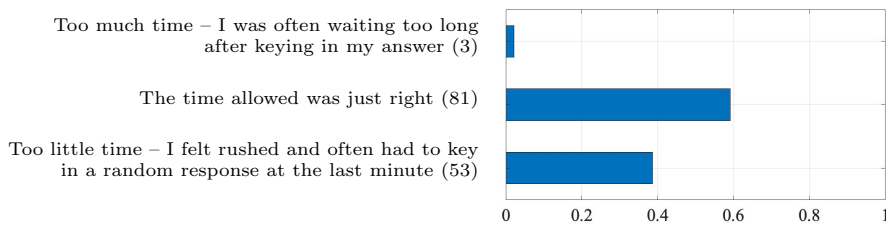
None (30)

One class (24)

More than one class (83)

0   0.2   0.4   0.6   0.8   1
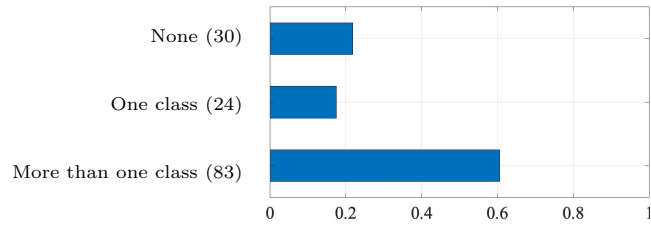
Q2. Overall, do you find that the iClicker questions have helped you to better understand the material presented in lectures?

Yes (116)

No (21)
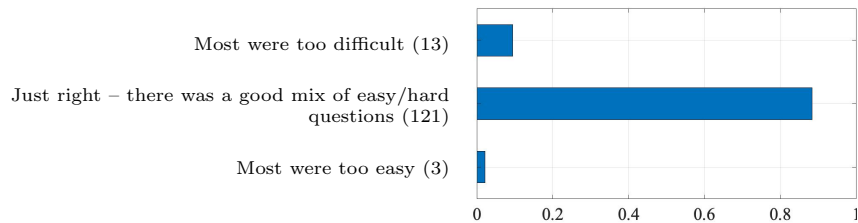
0   0.2   0.4   0.6   0.8   1

Q3. Pick the statement that best describes your experience with the iClicker questions.

They are good practice to make sure I understand concepts from lectures (112)

I often get the wrong answer and am left confused, even after the correct answer is discussed (16)

I don't get anything out of it and would rather focus on plowing through the lecture material (9)

0   0.2   0.4   0.6   0.8   1

Q4. Can you comment on the amount of time permitted to answer each question?

Too much time – I was often waiting too long after keying in my answer (3)

The time allowed was just right (81)

Too little time – I felt rushed and often had to key in a random response at the last minute (53)

0   0.2   0.4   0.6   0.8   1

Q5. Can you comment on the average level of difficulty of the questions?

Most were too difficult (13)

Just right – there was a good mix of easy/hard questions (121)

Most were too easy (3)

0   0.2   0.4   0.6   0.8   1

Q6. Do you have any preference regarding how the iClicker grade should be computed?
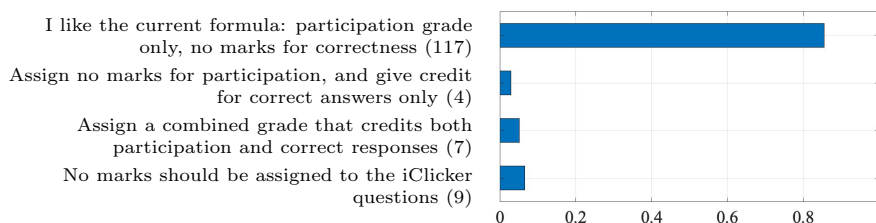
Q7. Assuming that some marks are assigned to completing the iClicker questions, what is your opinion of the portion 3% allocated to your final grade?
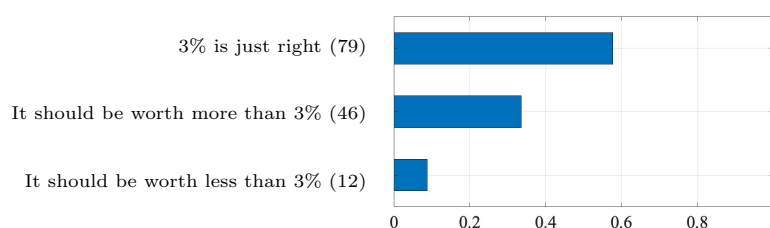
Q8. Rate your level of agreement with statements A-J on the following scale:

– Strongly Disagree, Disagree, Neutral, Agree, Strongly Agree –

A. [raise-hand] When a question is asked, I like the iClicker response option better than asking students to raise their hands to indicate their answer.

B. [volunteer] When a question is asked, I like the iClicker response option better than having an open invitation for any student to volunteer an answer.

C. [positive-break] I think iClickers provide a positive break from lectures.

D. [engaged] I feel more involved and engaged in class when we use iClickers.

E. [participation] I come to class more often because iClickers are used to track participation.

F. [retain-info] I retain more information from lecture when iClickers are used.

G. [distraction] I think iClickers distract from the learning process. ✱

H. [connection] I feel more connected to my classmates when we use iClickers.

I. [insights] iClickers help me to discover more insights into the course material.

J. [attention] It is easier to pay attention in lectures when iClickers are used.

Note that statement G (marked ✱) is the only one where a response of "strongly agree" has a *negative* connotation, and so this bar should be interpreted in the opposite manner to the others. This was done to verify that students were indeed carefully reading and applying the Likert scale to each

statement.
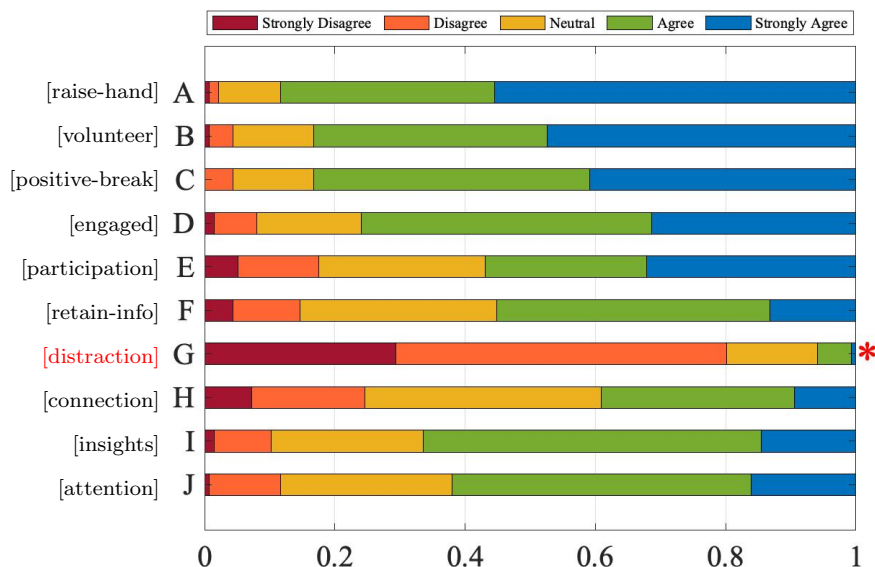
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