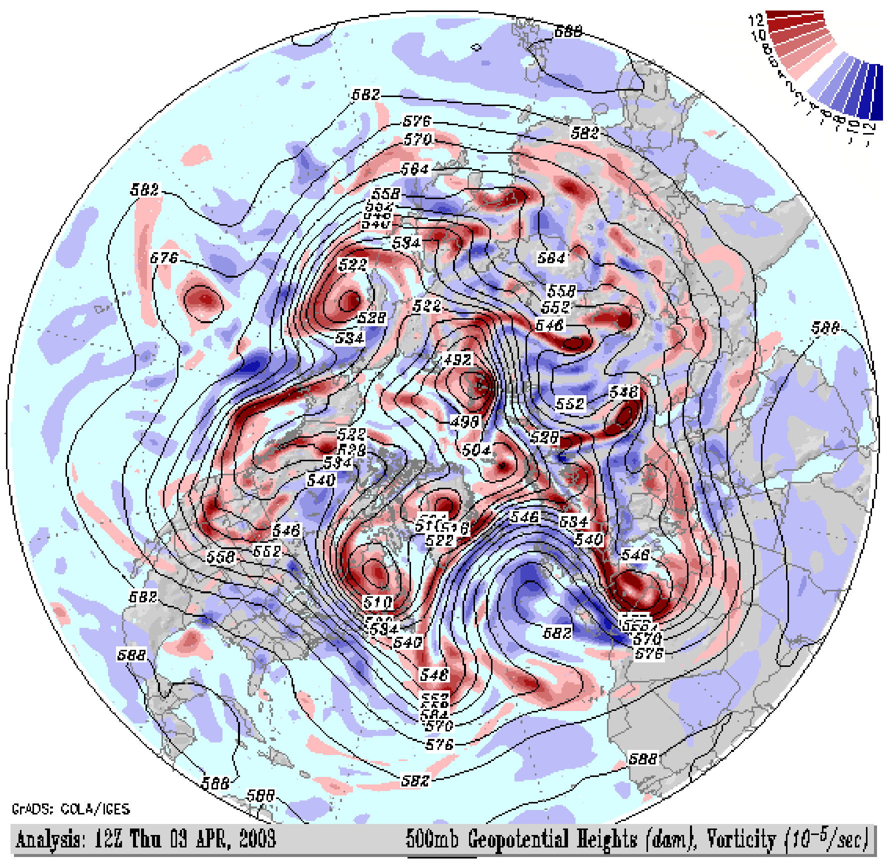
A Uniform PV Framework for Balanced Vortex Dynamics

- ▷ balanced models for a well-mixed troposphere
- ▷ tropopause- & surface-driven vortex dynamics



- ▷ Dave Muraki (Simon Fraser University)
- ▷ Greg Hakim (University of Washington)
- ▷ Chris Snyder (NCAR Boulder)

Midlatitude Vortex Dynamics: Some Questions

What is the mechanism behind the observed asymmetry of cyclones & anticyclones?

- ▷ localized, intense cyclones versus broad, weak anticyclones
- ▷ similar asymmetry observed for small-scale, upper-level vorticity disturbances; Hakim (2000)

What are the reasons for the differences in asymmetries seen in vortex simulations?

- ▷ previous simulations favoring organization of anticyclonic vorticity:
  - ⟶ rotating shallow water: Polvani, McWilliams etal. (1994)
  - ⟶ 3D periodic balance equations: Yavneh, McWilliams etal. (1994)
- ▷ finite Rossby number effects favor organization of cyclones:
  - ⟶ surface potential temperature dynamics (sQG^{+1}); Muraki, Hakim, Snyder (2002)

What is the rôle of the tropopause in the organization of upper-level vorticity?

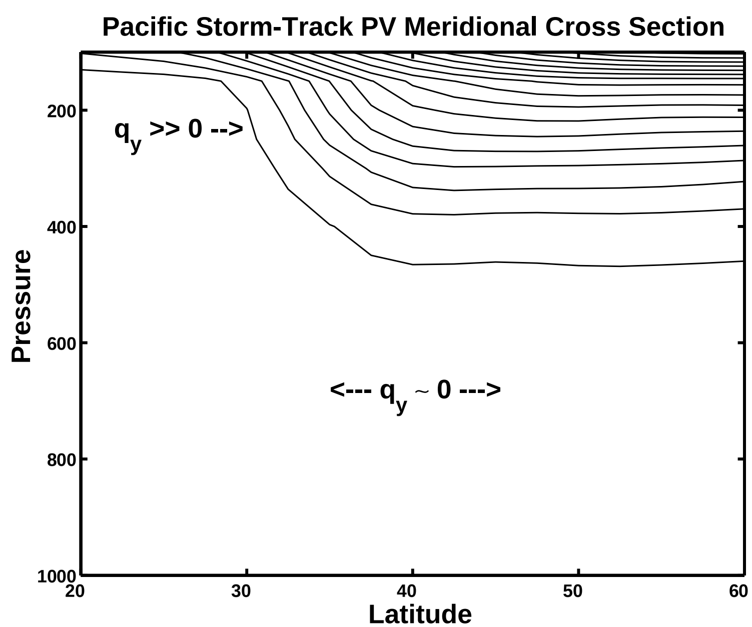
How does the depth of the troposphere influence vortex organization & dynamics?

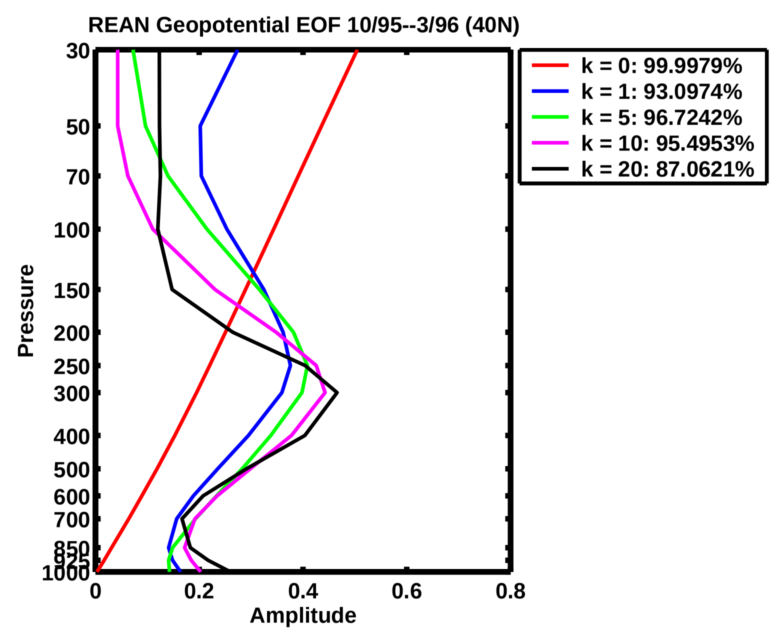
- ▷ with or without baroclinic instability, or in polar regions without strong jetstream

Vertical Structure of the Troposphere

Zonal Mean PV & Geopotential

- ▷ contours of zonal mean PV with latitude and height
- ▷ vertical profiles of Fourier amplitude of mean geopotential with zonal wavenumber k





- ▷ well-mixed PV in troposphere; disturbance amplitudes peak at the tropopause

Surface Quasigeostrophy

Quasigeostrophic Dynamics with Uniform PV

- ▷ baroclinic instability & Eady waves; Hoskins (1975/1976)
- ▷ wave interactions & turbulence; Blumen (1978)
- ▷ spectral turbulence; Pierrehumbert etal. (1994)
- ▷ dynamics & decaying turbulence; Held etal. (1995)

Dynamics of Surface Potential Temperature (θ^s)

- ▷ rigid surface at $z = 0$: surface potential temperature $\theta^s(x, y; t) = \theta(x, y, 0; t)$
- ▷ geostrophy: $v = \Phi_x \quad ; \quad u = -\Phi_y \quad ; \quad \theta = \Phi_z$
- ▷ inversion of uniform (zero) PV:

$$\nabla^2 \Phi = q = 0 \quad \text{with surface BC} \quad \Phi_z(x, y, 0; t) = \theta^s(x, y; t)$$

- ▷ surface advection:

$$\theta^s_t + u^s \, \theta^s_x + v^s \, \theta^s_y = 0$$

- ▷ dynamics are driven solely by surface advection

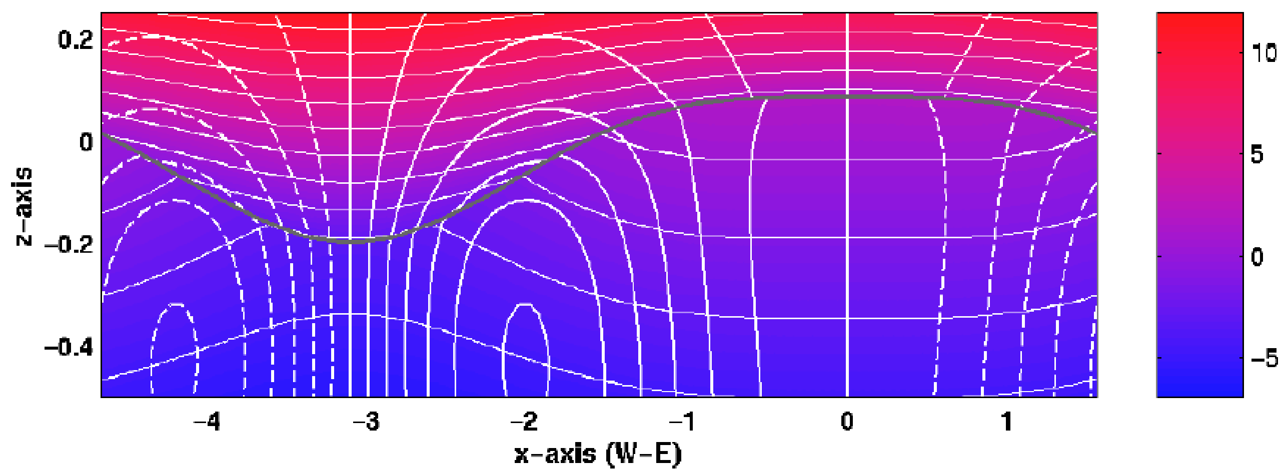
  → rôle of PV inversion is to determine surface winds (u^s, v^s) from θ^s

  → finite Rossby number corrections to surface winds, sQG^{+1}; Muraki, Hakim, Snyder (2002)

Tropopause as an Upper-Level Interface

A Two-PV Fluid Model for the Tropopause

- ▷ troposphere (low uniform PV) & stratosphere (high uniform PV)
- ▷ tropopause as dynamic interface between two-sQG fluids
  - ⟶ tropopause Eady wave; Rivest etal. (1992)
  - ⟶ tropopause dynamics; Juckes (1994)
- ▷ finite Rossby number corrections to Eady edge wave (sQG^{+1}); Muraki, Hakim (2001)
  - ⟶ ratio of stratospheric-to-tropospheric Burger numbers, $B^s/B^t = 4$



- ▷ cyclonic: intense, localized downward deflection & anticyclonic: weak, broad upward deflection
- ▷ disturbances decay away from tropopause (infinite fluid above & below)

sQG & The Rigid Tropopause Limit

A Simple Model for the Upper-Level Troposphere

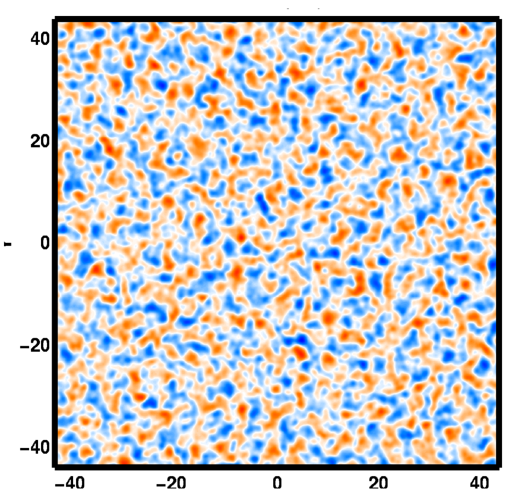
- ▷ limit as $B^s/B^t \rightarrow \infty$:

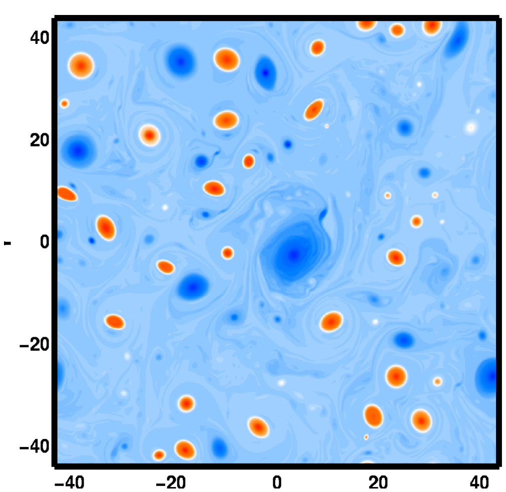
  $\longrightarrow$ sQG theory for troposphere ($z \leq 0$) only

  $\longrightarrow$ rigid tropopause boundary, $w^s \rightarrow 0$
- ▷ basic framework for understanding balanced disturbances localized at the tropopause

Organization of Vortices

- ▷ freely decaying turbulence from random initial vorticity; Muraki, Hakim, Snyder (2002)
- ▷ finite Rossby number corrections produce cyclone/anticyclone asymmetry





Vertical Structure of sQG

Inversion of Uniform PV

- ▷ for sQG below a rigid tropopause ($z \leq 0$)
- ▷ Fourier solution of the sQG streamfunction

$$\Phi(x, y, z; t) = \int_{-\infty}^{+\infty} \hat{\theta}^t(k, l; t) \left\{ \frac{e^{mz}}{m} \right\} e^{i(kx+ly)} \, dk \, dl$$

  $\longrightarrow$ Fourier transform of surface (tropopause) potential temperature: $\hat{\theta}^t(k, l; t)$

  $\longrightarrow$ inversion of Laplacian: $m = \sqrt{k^2 + l^2}$

- ▷ each Fourier mode decays exponentially as $z \longrightarrow -\infty$

  $\longrightarrow$ larger scales extend deeper into troposphere (smaller m)

  $\longrightarrow$ small scales are more localized to tropopause (larger m)

Computational Efficiency of sQG Fourier Inversion

- ▷ <u>resolution of vertical structure is exact for given horizontal discretization</u>
- ▷ only 2D FFTs required to evolve 3D tropospheric flow
- ▷ finite Rossby number corrections also computed with 2D efficiencies

Uniform PV Thinking

sQG Advantages for Understanding Dynamics

- ▷ rotating (f-plane), stratified fluid near an interface or surface
- ▷ balanced dynamics in zero Rossby number limit
- ▷ more faithful to continuously stratification than (barotropic) shallow water models
- ▷ extension to finite Rossby number corrections

Dynamics Beyond sQG

- ▷ finite tropospheric depth: passive bottom surface with $\theta^b = 0$
- ▷ two surfaces ($2sQG$): active top and bottom surfaces θ^t & θ^b

  $\longrightarrow$ barotropic alignment of top & bottom vortices at larger scales

  $\longrightarrow$ baroclinic instability when background shear included

- ▷ tropopause: stratospheric fluid above, moving interface at $z = \eta(x, y; t)$
- ▷ free-surface: unstratified fluid above, moving interface at $z = \eta(x, y; t)$

  $\longrightarrow$ continuously stratified analog to shallow water

Finite Tropospheric Depth Dynamics

Inversion of Uniform PV

- rigid tropopause at $z = H$, isentropic ground at $z = 0$
- Fourier solution of the 3D streamfunction ($m = \sqrt{k^2 + l^2}$)

$$\Phi(x, y, z; t) = \int_{-\infty}^{+\infty} \hat{\theta}^t(k, l; t) \left\{ \frac{\cosh mz}{m \sinh mH} \right\} e^{i(kx+ly)} \, dk \, dl$$

$\longrightarrow$ Fourier transform of tropopause potential temperature: $\hat{\theta}^t(k, l; t)$

Large & Small Scale Asymptotic Limits

- Fourier transform of surface ($z = H$) streamfunction, $\hat{\Phi}^t$

$$\hat{\Phi}^t(x, y; t) = \hat{\theta}^t(k, l; t) \left\{ \frac{1}{m \tanh mH} \right\} \sim \begin{cases} \dfrac{\hat{\theta}^t}{m} & mH \text{ large} \\[2ex] \dfrac{\hat{\theta}^t}{m^2 H} & mH \text{ small} \end{cases}$$

$\longrightarrow$ horizontal scales small relative to depth invert as sQG (mH large)

$\longrightarrow$ larger horizontal scales large invert as barotropic vorticity (mH small)

- on the large scales, $-\theta^t/H$ evolves like barotropic vorticity dynamics

Two Surface Dynamics

2sQG Inversion of Uniform PV

- ▷ rigid ground/tropopause surfaces at $z = 0, H$
- ▷ Fourier solution of the 3D streamfunction ($m = \sqrt{k^2 + l^2}$)

$$\Phi(x, y, z; t) = \int_{-\infty}^{+\infty} \hat{\theta}^t(k, l; t) \left\{ \frac{\cosh mz}{m \sinh mH} \right\} e^{i(kx+ly)} \, dk \, dl + \int_{-\infty}^{+\infty} \hat{\theta}^b(k, l; t) \left\{ \frac{\cosh m(H - z)}{m \sinh mH} \right\} e^{i(kx+ly)} \, dk \, dl$$

$\longrightarrow$ Fourier transform of surface potential temperatures: $\hat{\theta}^t(k, l; t)$ & $\hat{\theta}^b(k, l; t)$

- ▷ $(\theta^b + \theta^t)/2H$ dynamically acts like the baroclinic flow component
- ▷ $\zeta = (\theta^b - \theta^t)/2H$ dynamically acts like barotropic vorticity

Large-Scale Dynamics act Barotropically

- ▷ t, b-mean streamfunction ($\bar{\Phi}$) over small mH wavenumbers:

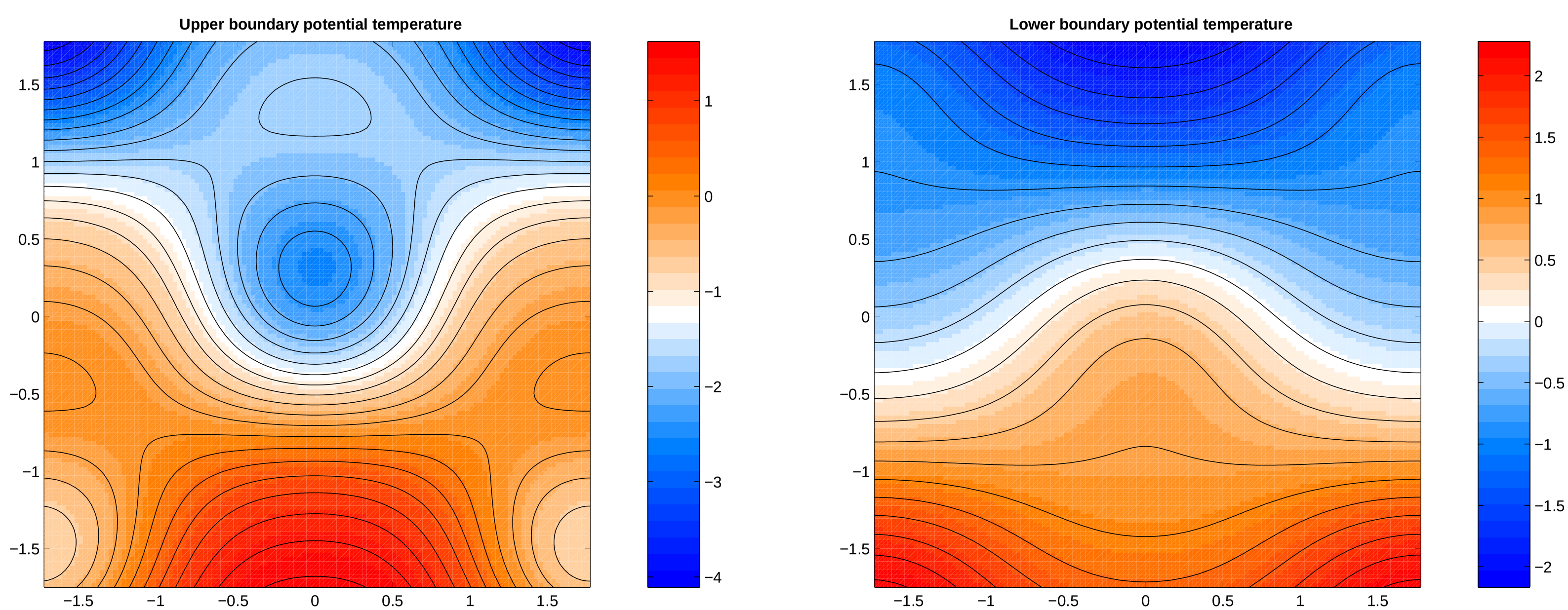
$$\bar{\Phi}(x, y; t) \approx \int_{-\infty}^{+\infty} \hat{\zeta}(k, l; t) \left\{ \frac{-1}{m^2} \right\} e^{i(kx+ly)} \, dk \, dl$$

- ▷ advection of barotropic component by mean wind: $\zeta_t + J(\bar{\Phi}, \zeta) = 0$

Two-Surface Edge Wave

Finite Rossby Number Corrections

- nonlinear edge wave solution with simple Eady shear, correct to $O(\mathcal{R})$
- square wave $k = l = 1$, vertical mode number $m = \sqrt{k^2 + l^2} = 2.5$
- beyond short-wave stability criterion: $m > m_c \approx 2.399$
- upper-level cyclone asymmetry for $\mathcal{R} = 0.1$
- nonlinear wavespeed same as neutral linear edge waves



Free-Surface Dynamics

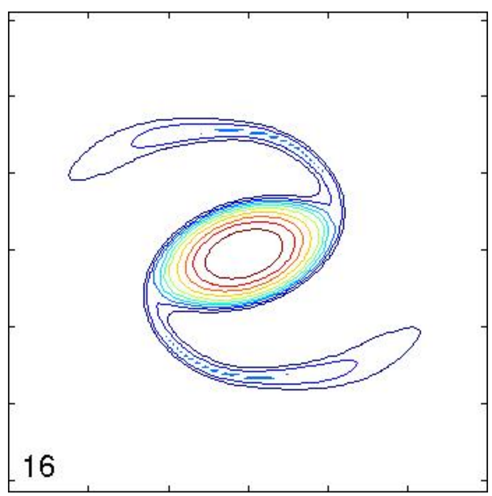
Uniform PV Inversion (with R Tulloch)

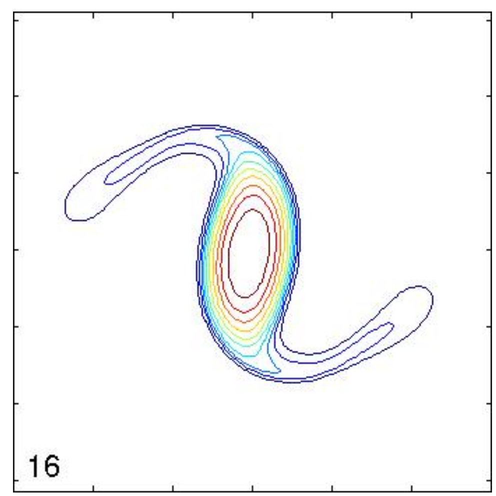
- moving free-surface at $z = \mathcal{R}h(x, y; t)$
- total surface potential temperature, $\theta^s(x, y; t) = h(x, y; t) + \theta(x, y, \mathcal{R}h(x, t; t), t)$
- surface BCs: kinematic conditions with continuity of potential temperature and pressure
- Fourier solution of the 3D streamfunction ($m = \sqrt{k^2 + l^2}$)

$$\Phi(x, y, z; t) = \int_{-\infty}^{+\infty} \hat{\theta}^s(k, l; t) \left\{ \frac{1}{m + \sigma^{-1}} \right\} e^{i(kx+ly)} \, dk \, dl$$

$\longrightarrow$ surface value of potential temperature is $-\sigma$

- surface anticyclones: sQG^{+1} versus fsQG^{+1} (slower rotation, less axisymmetrization)





Summary

Dynamics of Uniform PV Layers

- ▷ significant part of tropospheric dynamics are strongly influenced by tropopause and ground
- ▷ simple formulation for understanding rotating, stratified flows dominated by surfaces/boundaries
- ▷ two-surface dynamics naturally embeds large- and small-scale limiting cases:
  - ⟶ large-scale barotropic vorticity dynamics
  - ⟶ small-scale surface-trapped dynamics
- ▷ moving interface formulation naturally includes:
  - ⟶ tropopause dynamics
  - ⟶ free-surface dynamics, as a continuously-stratified shallow-water analog

Emerging Applications to Tropospheric Dynamics

- ▷ origins & evolution of vortex asymmetries
- ▷ finite Rossby number effects in baroclinic instability
- ▷ dynamics of tropopause disturbances
- ▷ stratified *shallow water* dynamics