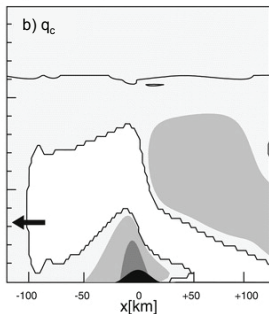


Internal gravity waves in a saturated, moist-neutral atmosphere

- ▷ upstream wave of subsidence in start-up for moist topographic flow
- ▷ asymmetric buoyancy response of cloud-free air to up/down displacements
- ▷ cloud-edge motion by a gravity-wave shock



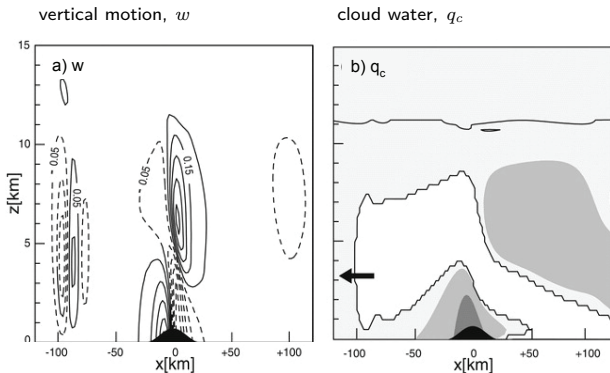
- ▷ David J Muraki (Simon Fraser Univ) & Rich Rotunno (NCAR)



Start-Up for Moist 2D Flow Over Topography

Saturated, Moist-Neutral Atmosphere

- ▷ WRF: full-physics, topographic lower boundary (Miglietta/Rotunno, 2005)
- ▷ waves trapped by tropopause boundary (Keller, et al., 2012)



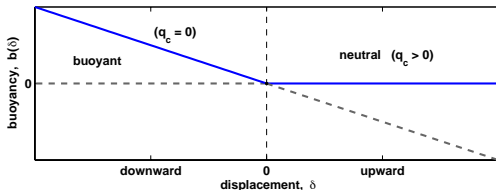
- ▷ why is there an upstream propagating wave of subsidence → de-saturated, cloud-free air?

Desaturation of Cloud-Free, Moist-Neutral Air

Primary Cause: Asymmetry to Up/Down Displacements

- ▷ saturated & cloud-free ($q_c = 0$) at zero displacement ($\delta = 0$)
- ▷ UPward displacement ($\delta > 0$):
 - ▷ release of latent heat by condensation \rightarrow neutral buoyancy (reversible)
 - ▷ \approx constant θ_e , Clausius-Clapeyron thermodynamics
- ▷ DOWNward displacement ($\delta < 0$):
 - ▷ unsaturated (no available cloud water) \rightarrow stratified buoyancy
 - ▷ constant N^2

Buoyancy as a Function of Displacement, $b(\delta)$

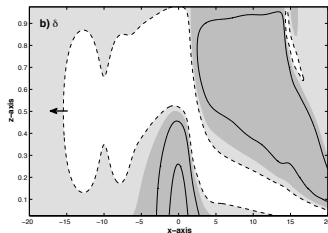
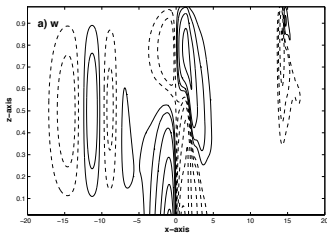


- ▷ critical sensitivity of wave propagation to downward displacement

A Hierarchy of Models, Part I

Dynamics of Asymmetric Buoyancy

- ▷ 2D “linear” Boussinesq primitive equations & linearized topographic boundary displacement, δ
- ▷ vertical motion, w



- ▷ 2D vorticity/streamfunction: $u = \psi_z$, $w = -\psi_x$

$$(\eta_t + U\eta_x) + (b(\delta))_x = 0$$

$$(\delta_t + U\delta_x) + \psi_x = 0$$

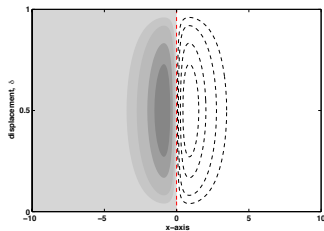
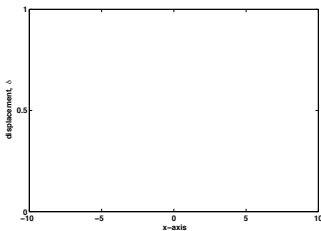
$$\psi_{xx} + \psi_{zz} = \eta$$

- ▷ the upstream propagating wave of subsidence persists ...

A Hierarchy of Models, Part II

Dynamics of Asymmetric Buoyancy

- ▷ 1D “linear” single vertical-mode & ($U = 0$) initial displacement
 - ▷ vertical motion, $w \rightarrow w(x, 0) \sin z$ displacement, $\delta \rightarrow \delta(x, 0) \sin z$



- ▷ 2D vorticity/streamfunction: $w = -\psi_x$

$$\eta_t + (b(\delta))_x = 0$$

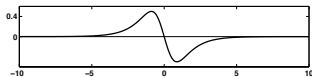
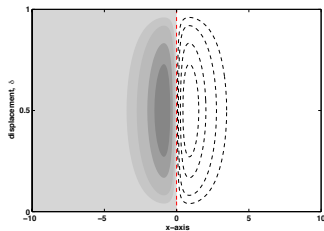
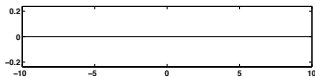
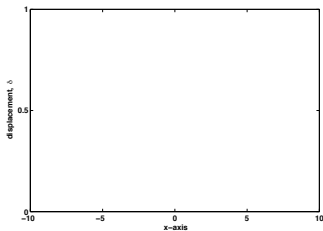
$$\delta_t + \psi_x = 0$$

$$\psi_{xx} - \psi = \eta$$

A Hierarchy of Models, Part II

Dynamics of Asymmetric Buoyancy

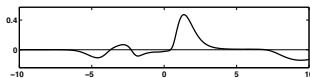
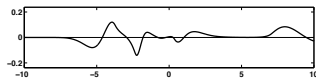
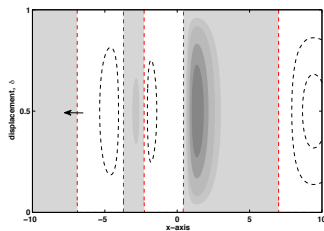
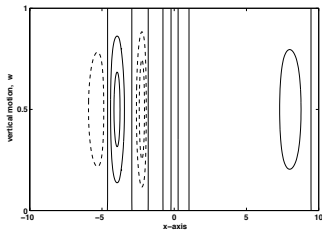
- ▷ 1D “linear” single vertical-mode & ($U = 0$) initial displacement
- ▷ vertical motion, $w \rightarrow w(x, 0) \sin z$ displacement, $\delta \rightarrow \delta(x, 0) \sin z$



A Hierarchy of Models, Part II

Dynamics of Asymmetric Buoyancy

- ▷ 1D “linear” single vertical-mode & ($U = 0$) initial displacement
- ▷ vertical motion, $w \rightarrow w(x, t) \sin z$ displacement, $\delta \rightarrow \delta(x, t) \sin z$

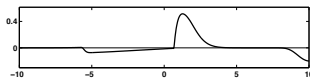
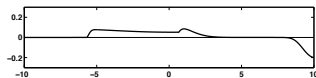
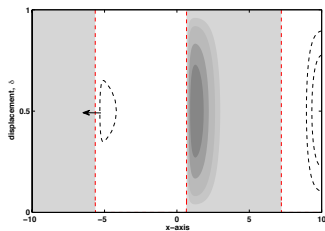
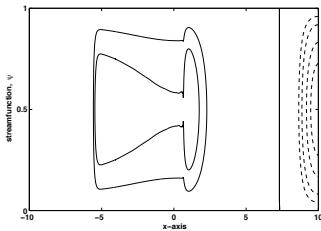


- ▷ the upstream propagating wave of subsidence yet persists ...

A Hierarchy of Models, Part III

Dynamics of Asymmetric Buoyancy

- ▷ 1D “linear” single vertical-mode, **hydrostatic** & ($U = 0$) initial displacement
- ▷ streamfunction, $\psi \rightarrow \psi(x, t) \sin z$ displacement, $\delta \rightarrow \delta(x, t) \sin z$

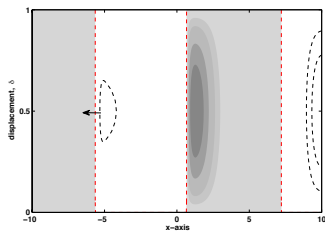
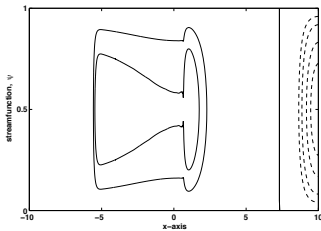


- ▷ ... & the upstream propagating wave of subsidence stubbornly remains

A Hierarchy of Models, Part III

Dynamics of Asymmetric Buoyancy

- ▷ 1D “linear” single vertical-mode, $U = 0$, **hydrostatic** & initial displacement
- ▷ streamfunction, $\psi \rightarrow \psi(x, t) \sin z$ displacement, $\delta \rightarrow \delta(x, t) \sin z$



- ▷ PDE conservation laws for ψ and δ

$$-\psi_t + (b(\delta))_x = 0$$

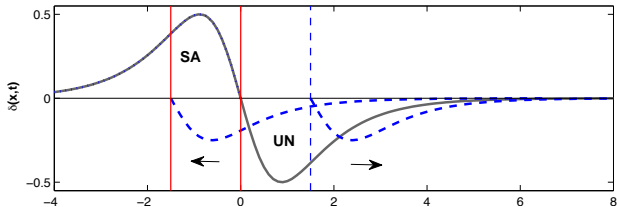
$$\delta_t + \psi_x = 0$$

- ▷ so now ... why is there an upstream propagating wave of subsidence?

Two Buoyancy Regimes

Saturated, No Propagation vs Unsaturated Gravity Waves

- ▷ hydrostatic waves have a characteristic speed
- ▷ saturated (SA): upward displacement, $\delta > 0$
 - ▷ wavespeed = 0 \rightarrow no buoyancy waves
- ▷ unsaturated (UN): downward displacement, $\delta < 0$
 - ▷ wavespeed = $\pm c \rightarrow$ wave equation ($\delta_{tt} - c^2 \delta_{xx} = 0$)

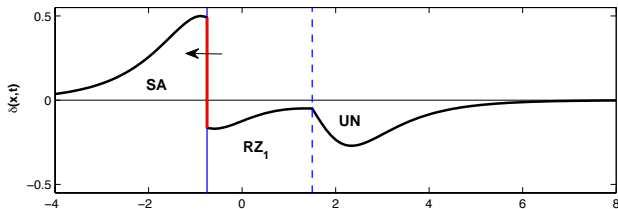


- ▷ what happens at the upstream "collision" of SATurated & UNsaturated air ... ?

Two Buoyancy Regimes

Saturated, No Propagation vs Unsaturated Gravity Waves

- ▷ hydrostatic waves have a characteristic speed
- ▷ saturated (SA): upward displacement, $\delta > 0$
 - ▷ wavespeed = 0 \rightarrow no buoyancy waves
- ▷ unsaturated (UN): downward displacement, $\delta < 0$
 - ▷ wavespeed = $\pm c \rightarrow$ wave equation ($\delta_{tt} - c^2 \delta_{xx} = 0$)

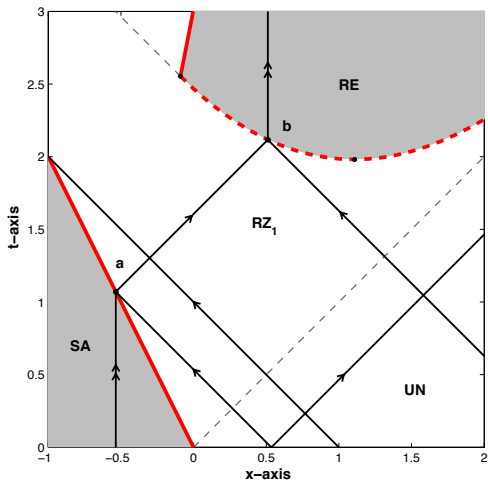


- ▷ ... an upstream propagating shock \rightarrow wave of subsidence
 - ▷ speed $\approx -c/2$ by Rankine-Hugoniot (consistent with MR, 2005)

Hydrostatic Solution by Method of Characteristics

Characteristics, Shocks & $\delta = 0$ Discontinuities

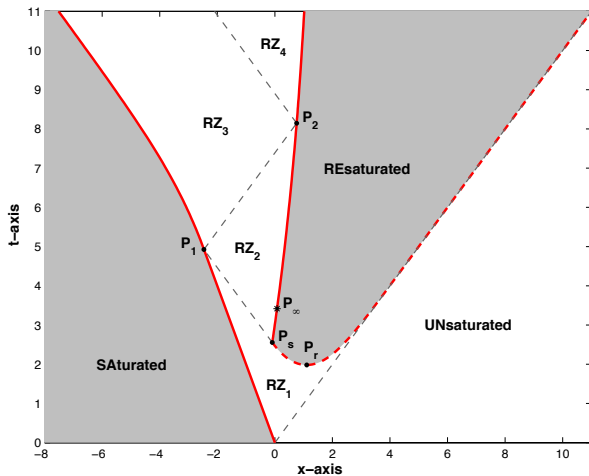
- ▷ space-time (Hovmüller) map of SATurated & UNSaturated regions



Hydrostatic Solution by Method of Characteristics

Characteristics, Shocks & $\delta = 0$ Discontinuities

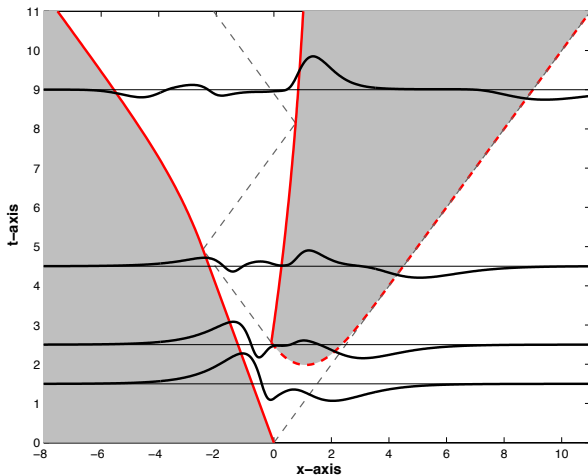
- ▷ space-time (Hovmüller) map of SATurated & UNSaturated regions



Nonhydrostatic Waves vs Hydrostatic Regions

Upstream Wave of Subsidence

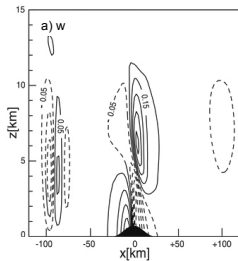
- ▷ 1D nonhydrostatic displacement, $\delta(x, t)$: wave regions & shock-like transitions



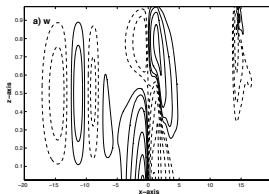
In Closing

Dynamics of Saturated, Moist-Neutral Air

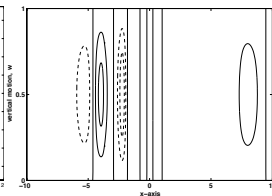
- ▷ asymmetric vertical motions about cloud-free state
 - ▷ topographic start-up: upstream wave of subsidence as a (hydrostatic) propagating shock
 - ▷ gravity-wave shock: a new mechanism for the motion of cloud-edges (in moist-neutral air)
-
- ▷ upstream wave of downward vertical motion
 - ▷ WRF



2D



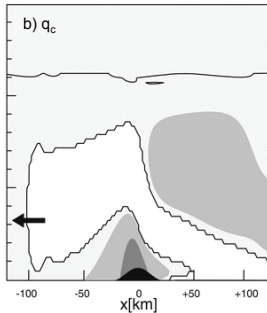
1D nonhydrostatic



In Closing

Dynamics of Saturated, Moist-Neutral Air

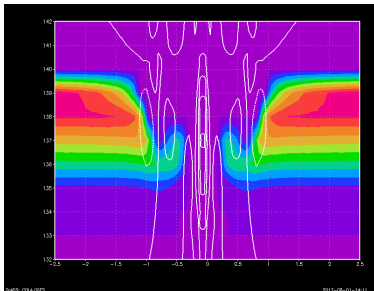
- ▷ asymmetric vertical motions about cloud-free state
- ▷ topographic start-up: upstream wave of subsidence as a (hydrostatic) propagating shock
- ▷ gravity-wave shock: a new mechanism for the motion of cloud-edges



In Closing

Dynamics of Saturated, Moist-Neutral Air

- ▷ asymmetric vertical motions about cloud-free state
- ▷ topographic start-up: upstream wave of subsidence as a (hydrostatic) propagating shock
- ▷ gravity-wave shock: a new mechanism for the motion of cloud-edges
- ▷ hole-punch clouds (Heymtsfield, et al, 2010/11)



- ▷ WRF simulations: Morrison & Thompson (NCAR)