Vortex Dynamics on the Tropopause

- ▷ vorticity dynamics for rotating, stratified flow
- ▷ symmetry-breaking in the atmosphere

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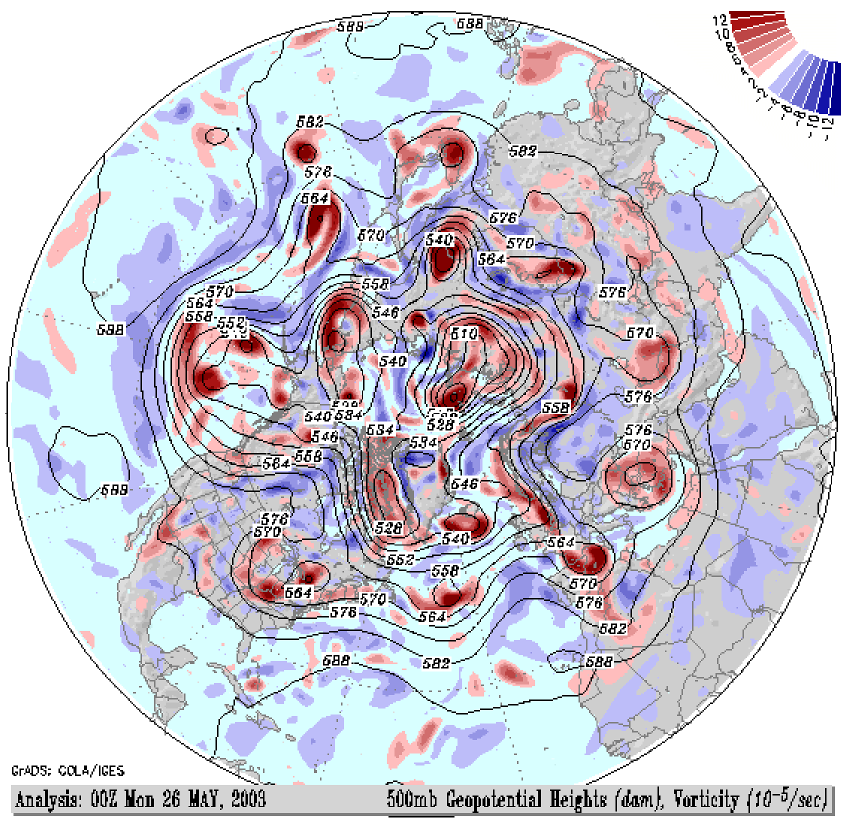

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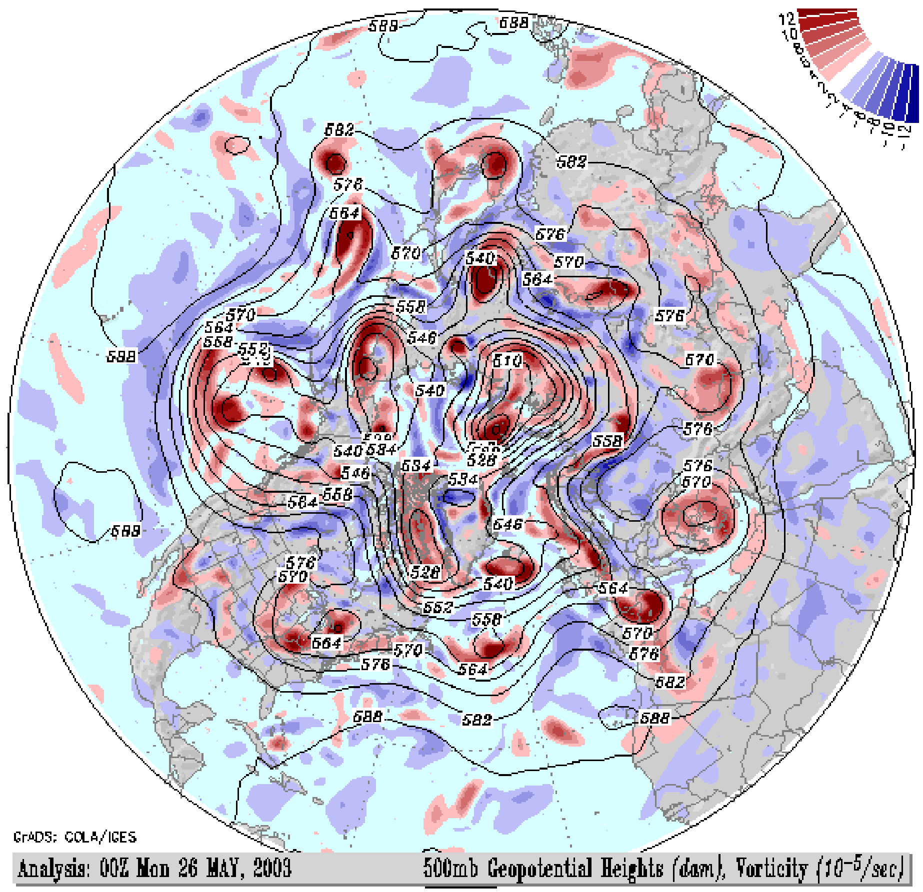
Asymmetric Dynamics of Midlatitude Vortices

- localized, intense cyclones (low pressure) versus broad, weak anticyclones (high pressure)
- contours of geopotential are streamlines → midlatitude jetstream



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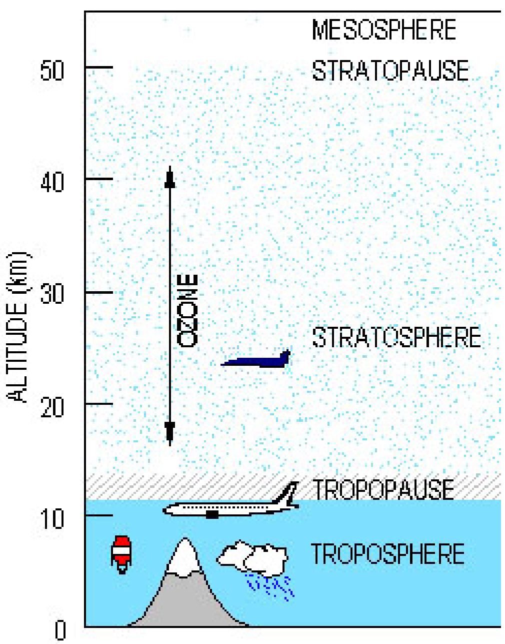
Symmetry-Breaking & Atmospheric Structure

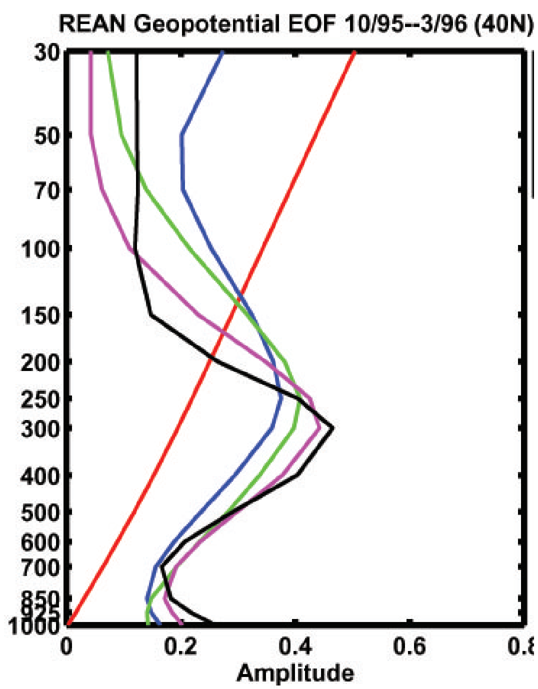
What is the mechanism behind the observed asymmetry of cyclones & anticyclones?

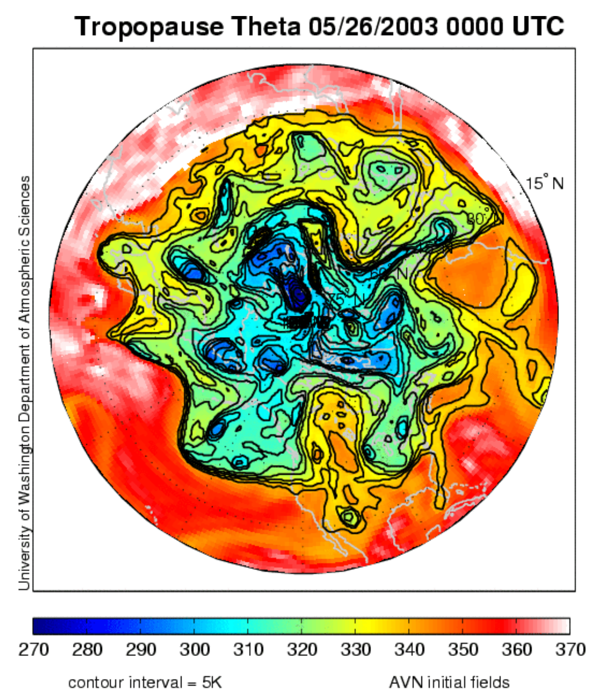
- ▷ understanding our 3D atmosphere in terms of a 2D dynamics?

Tropopause-Based Dynamics

- ▷ troposphere: lowest, weather-containing layer of the atmosphere ($\approx 0 - 10$km)
- ▷ tropopause: troposphere/stratosphere interface $\longrightarrow$ organizing level for dynamics?







Vorticity Dynamics

2D Euler: Vorticity & Streamfunction

- 2D flow gives advection of vorticity, $\zeta(x, y; t)$:

$$\frac{D\zeta}{Dt} = \zeta_t + u\zeta_x + v\zeta_y = 0 \quad ; \quad \nabla^2\Phi = \zeta$$

- velocity streamfunction from 2D vorticity inversion:

$$u = -\Phi_y \quad ; \quad v = \Phi_x$$

3D Quasigeostrophy: Potential Vorticity (PV) & QG Streamfunction

- QG approximation gives horizontal advection of PV, $q(x, y, z; t)$:

$$\frac{Dq}{Dt} = q_t + uq_x + vq_y = 0 \quad ; \quad \nabla^2\Phi = q$$

- QG streamfunction from 3D PV inversion:

$$u = -\Phi_y \quad ; \quad v = \Phi_x$$

  $\longrightarrow$ horizontally non-divergent flow

- thermodynamic variable: potential temperature, $\theta = \Phi_z$

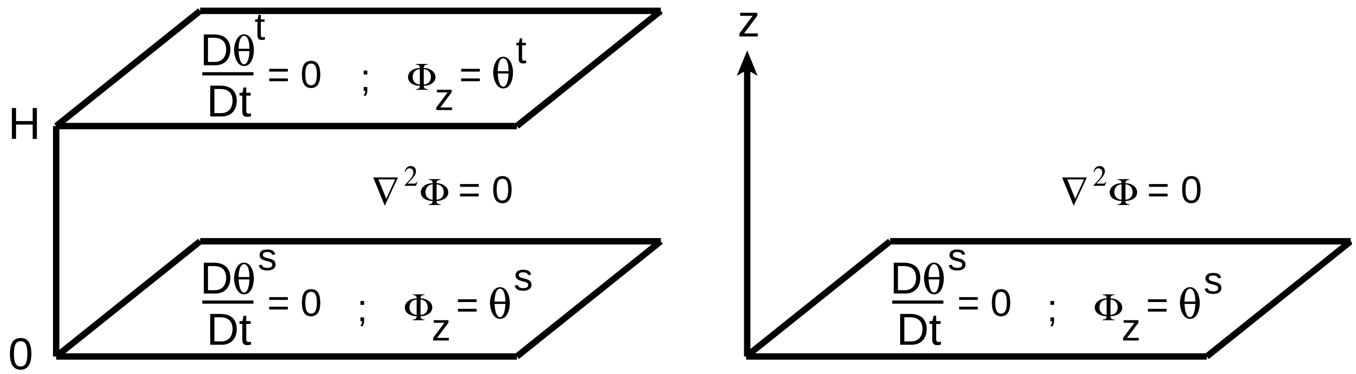
- QG approximation: zero Rossby number limit of rotating, stratified flow

  $\mathcal{R} = U/fL$ where f is the Coriolis frequency

Zero PV Surface Dynamics

Well-Mixed Troposphere Assumption $\Rightarrow q \equiv 0$

- QG streamfunction determined by surface/tropopause BCs



Surface Quasigeostrophy (sQG)

- semi-infinite fluid ($z \geq 0$), periodic in x, y, decay as $z \to +\infty$
- 3D inversion of zero PV:

$$\nabla^2\Phi = 0 \quad ; \quad \Phi_z(z=0) = \theta^s \quad ; \quad \Phi(z \to +\infty) = 0$$

- 2D advection of surface potential temperature, θ^s:

$$\frac{D\theta^s}{Dt} = \theta^s_t + u\,\theta^s_x + v\,\theta^s_y = 0 \quad ; \quad u = -\Phi_y(z=0) \quad ; \quad v = \Phi_x(z=0)$$

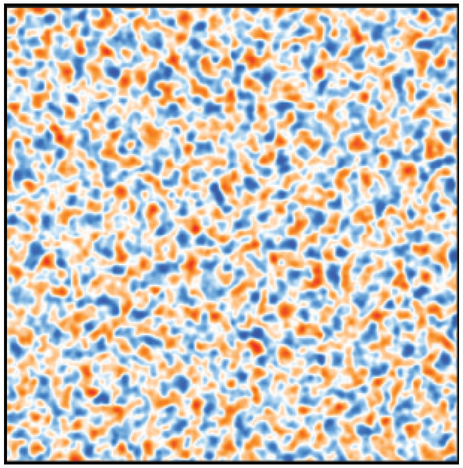
- sQG *interface* as model for tropopause: Rivest, et.al. (1992); Juckes (1994)

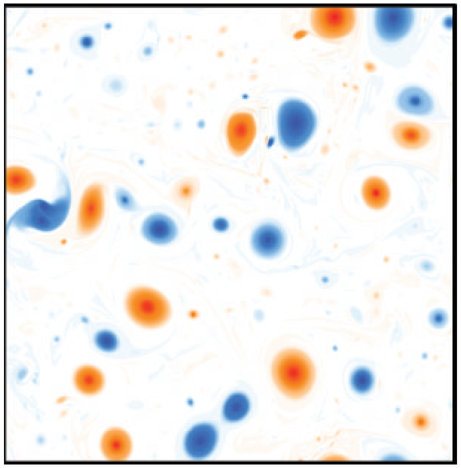
A Question of Asymmetry

Geostrophic Turbulence: unforced, decaying vortex dynamics

- ▷ surface QG ⇒ symmetric

  Pierrehumbert, et.al. (1994); Held, Pierrehumbert, Garner & Swanson (1995)





- ▷ 2D shallow water ⇒ weak anticyclonic bias at small Rossby number

  Polvani, McWilliams, Spall & Ford (1994)

- ▷ 3D periodic balance equations ⇒ weak anticyclonic bias at small Rossby number

  Yavneh, Shchepetkin, McWilliams & Graves (1997)

- ▷ <u>idealized rotating, stratified model which includes cyclone intensification?</u>

Primitive Equations (PE) for Rotating, Stratified Flow

Rotating (f-plane), Stratified (stable), Boussinesq Buoyancy, Hydrostatic

$$
\begin{aligned}
u_x + v_y \quad + \mathcal{R}\, w_z &= 0 \\
\mathcal{R}\left\{\frac{Du}{Dt}\right\} - v &= -\phi_x \\
\mathcal{R}\left\{\frac{Dv}{Dt}\right\} + u &= -\phi_y \\
\delta^2\left\{\frac{Dw}{Dt}\right\} - \theta &= -\phi_z \\
\left\{\frac{D\theta}{Dt}\right\} + w &= 0
\end{aligned}
$$

- ▷ potential temperature: θ (cold = heavy ; warm = light)
- ▷ geopotential: ϕ (pressure)
- ▷ advection: $\dfrac{D}{Dt} = \dfrac{\partial}{\partial t} + u\,\dfrac{\partial}{\partial x} + v\,\dfrac{\partial}{\partial y} + \mathcal{R}\, w\,\dfrac{\partial}{\partial z}$
- ▷ 3D quasigeostrophy is zero Rossby number ($\mathcal{R} \to 0$) limit of the PE

Small Rossby Number Corrections

Next-Order Inversion

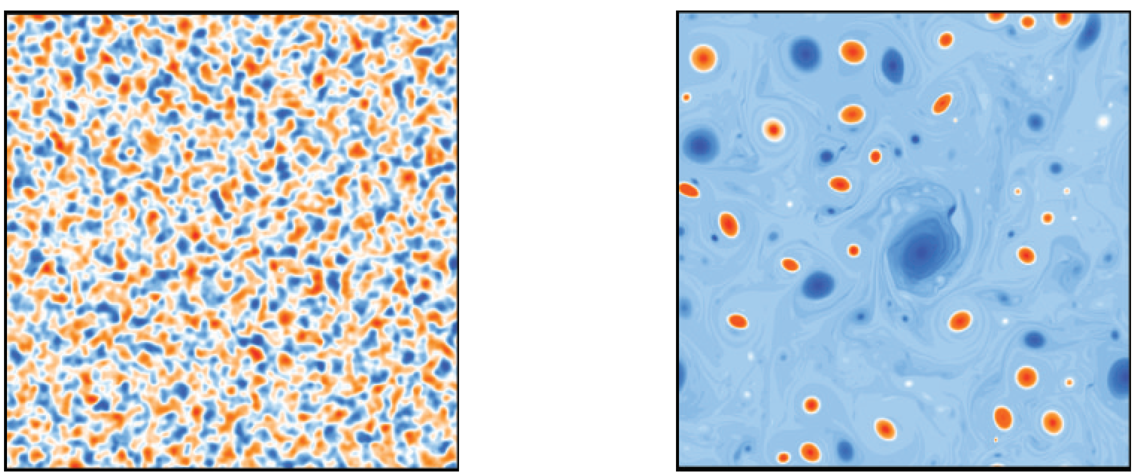
- small-$\mathcal{R}$ degenerate perturbation theory, resolve using Helmholtz representation

$$
\begin{aligned}
v &= \Phi_x - G_z \\
-u &= \Phi_y + F_z \\
\theta &= \Phi_z + G_x - F_y \\
\mathcal{R}\, w &= F_x + G_y
\end{aligned}
$$

- sequence of elliptic solves for: $\Phi \sim \Phi^0 + \mathcal{R}\Phi^1$; $F \sim \mathcal{R}F^1$; $G \sim \mathcal{R}G^1$

Asymmetric Organization of Vortices

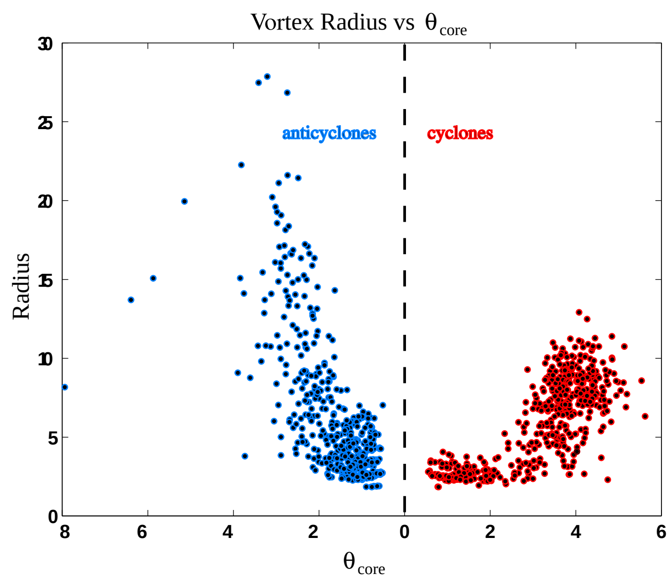
- freely-decaying turbulence from random, symmetric initial conditions; Hakim, Snyder, djm (2002)

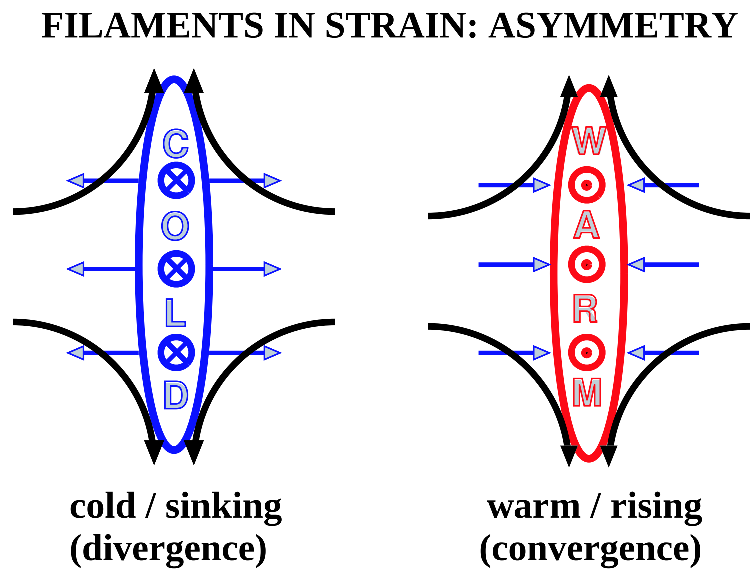


Mechanics of Vortex Asymmetry

Vortex Statistics

- scatterplot of strength versus size of cyclones & anticyclones





Asymmetry from Horizontally Divergent Flow

- frontogenesis: steeper-edged cyclones, discourages merger & filamentation
- frontolysis: broadly-spread anticyclones, encourages merger & filamentation
- surface cooling of mean θ: relative strengthening of cyclones
- filament roll-up instability: anticyclonic bias

Conclusions & Future Directions

Finite-Rossby Number Mechanisms for Asymmetry

- ▷ $O(\mathcal{R})$ horizontally divergent flow & implied vertical motion
  $\longrightarrow$ divergent flow events recently observed in tropopause data; Hakim
- ▷ vortex stretching relative to a surface
  $\longrightarrow$ not present in 3D-periodic dynamics
- ▷ net surface cooling
  $\longrightarrow$ shallow water dynamics preserve center of mass

Applications of Zero PV Surface Dynamics

- ▷ advection dynamics & elliptic inversions are computationally 2D
  $\longrightarrow$ finite-depth effects: recovers 2D vorticity dynamics at largest scales
  $\longrightarrow$ dynamic tropopause interface: comparison with tropopause observations
  $\longrightarrow$ free-surface boundary condition: does it more strongly recover shallow water dynamics?
  $\longrightarrow$ 2-surface dynamics: includes asymmetric baroclinic instability
  $\longrightarrow$ random forcing of sQG: emergence of vortices in the absence of jets

Exact Reformulation of PE

- three-potential representation: Φ, F, G

$$
\begin{aligned}
v &= \Phi_x - G_z \\
-u &= \Phi_y + F_z \\
\theta &= \Phi_z + G_x - F_y \\
\mathcal{R}\, w &= F_x + G_y
\end{aligned}
$$

- potential inversions

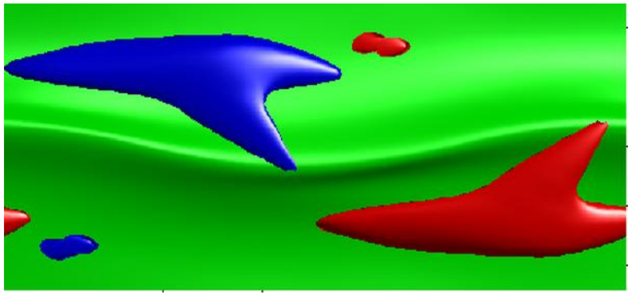
$$
\begin{aligned}
\nabla^2 \Phi &= q - \mathcal{R} \left\{ \nabla \cdot \left[\theta \left(\nabla \times \vec{\mathbf{u}}_H \right) \right] \right\} \\
\nabla^2 F &= \mathcal{R} \left\{ - \left(\frac{D\theta}{Dt} \right)_x + \left(\frac{Dv}{Dt} \right)_z \right\} \\
\nabla^2 G &= \mathcal{R} \left\{ - \left(\frac{D\theta}{Dt} \right)_y - \left(\frac{Du}{Dt} \right)_z \right\}
\end{aligned}
$$

- surface boundary conditions

$$
\mathcal{R}\, w^s = (F_x + G_y)^s \quad ; \quad \theta^s = (\Phi_z + G_x - F_y)^s
$$

- advection dynamics (interior & surface)

$$
\frac{Dq}{Dt} = 0 \qquad ; \qquad \frac{D\theta^s}{Dt} + w^s = 0
$$



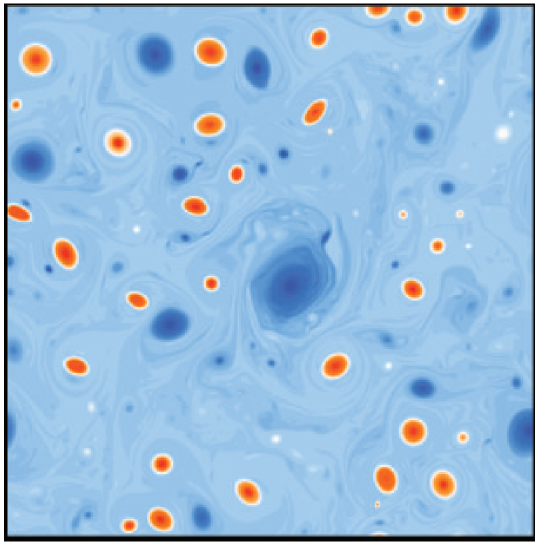
Turbulent Dynamics in Geophysical Flows

Dynamics in a World Driven by Turbulent Diffusion

- ▷ Esteban Tabak, New York University

Generation of Large-Scale Jets, Vortices & Layers from Near-Resonant Interactions of Fast & Slow Waves

- ▷ Leslie Smith, University of Wisconsin



The Coherence of Turbulence

- ▷ Fabian Waleffe, University of Wisconsin

Vortex Dynamics on the Tropopause

- ▷ Dave Muraki, Simon Fraser University