Models of Nonlinearity: Dynamics, Patterns & Waves

Diffusion and propagation are processes modelled by partial differential equations (PDEs) which are linear, and hence, also very well understood. Quantifying the effects of nonlinearity however, is much less straightforward since the usual tools of linear theory (e.g. Fourier theory, Greens functions) are not applicable. Despite the dearth of general theories, intuition about nonlinear ODEs and PDEs can be obtained from simple model equations through a combination of special solution methods, asymptotic & perturbation analyses and numerical computation.

The theme of the lectures will be two-fold. Mathematical techniques will be discussed within the context of simple nonlinear models, the scope of which will also serve as an introductory survey of nonlinear PDE phenomena. The simplest methods apply to weakly nonlinear PDEs and are appropriate for studying multiple end-states (bifurcation theory) and instabilities. A variety of asymptotic methods for nonlinear oscillations lead naturally to descriptions for the nonlinear modulation of waves. Strongly nonlinear spatial structures, like solitary waves and fronts, give insight into the self-organizing modes which often arise in the evolution of patterns. Lastly, Hamiltonian systems offer a bridge between the theories of ODE chaos and PDE complexity.

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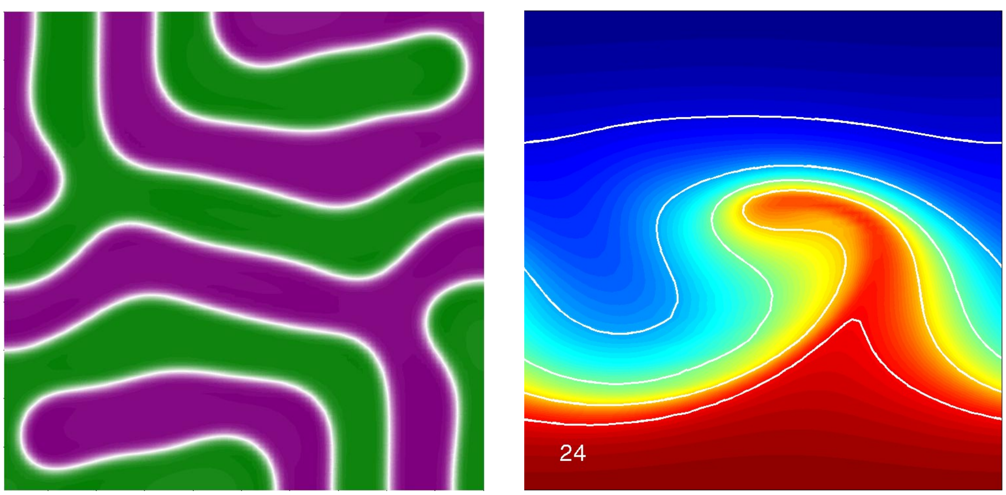

The image on the left is a labyrinthine pattern of concentration obtained from a model describing a realizable, albeit slightly esoteric, chemical reaction. The image on the right is a snapshot of the surface temperature from a model describing the development of atmospheric storms. In both of these cases, nonlinearity plays a significant role in determining the formation of the spatial structure.