MATH 817 ASSIGNMENT 2

DUE OCTOBER 15, 2009, IN CLASS

If your assignment must be late for any reason please notify me (by email, phone or in person) **before** the assignment is due. There will be no retroactive lates.

(1) (Isaacs problem 3.11). Write G'' for $(G')'$.
   (a) Suppose G'' is cyclic. Show that $G'' \subseteq Z(G')$.
   (b) Suppose in addition that G'/G'' is cyclic. Show that $G'' = 1$.

(2) (Isaacs problem 7.2). A subgroup $D \subseteq G = M \dot{\times} N$ is a *diagonal* subgroup if
$$D \cap M = 1 = D \cap N \quad \text{and} \quad DM = G = DN.$$
Show that G has a diagonal subgroup iff $M \cong N$.

(3) (Isaacs problem 7.11). Show that a finite nonabelian p-group cannot split over its center.

(4) This question refers to Isaacs Theorem 7.17 (attached for those without the book), and to our constructive definition of semidirect products from class.
   (a) Show that conditions (a) through (d) of Theorem 7.17 uniquely determine G.
   (b) Show that the G of Theorem 7.17 is isomorphic to $N \rtimes H$ as defined in class.

(5) (Isaacs problem 6.1) Let $P \subset S_n$ be a subgroup of prime order and suppose $x \in S_n$ normalizes but does not centralize P. Show that x fixes at most one point in each orbit of P.

(6) (Miller exercise 2.2) Show that in a free group two elements commute iff they are powers of a common element.