MATH 343, SPRING 2013
FINAL EXAM NOTES AND REVIEW QUESTIONS

Final notes

The final exam will be Wednesday April 17 from 3:30pm to 6:30pm in AQ5018. It will cover everything we have done. About 2/3 of the exam is on material since the midterm and the remaining 1/3 is on material from before the midterm.

There is some choice in which questions you complete; you will need to complete 12 questions. Some questions are more theoretical, some are more computational. Proofs from class are fair game, as are algorithms, and things from the homework. You won't have a computer so I can't ask for implementations, but I can ask for pseudocode for algorithms.

Final review questions

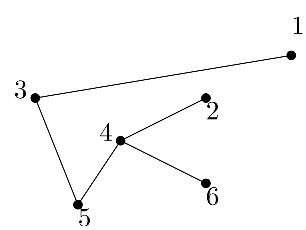
Here are some review questions.

(1) Let $\mathcal{T}$ be the class of rooted trees where every vertex has a number of children which either is 0 or is divisible by 2 but not 4.
   (a) Give a specification for $\mathcal{T}$.
   (b) Give an equation satisfied by the generating function $T(x)$.
   (c) If you wanted to find a formula for $[x^n]T(x)$ what technique could you use?

(2) Let $\mathcal{B}$ be the combinatorial class of binary strings with the property that every block of 0s is followed by an odd number of 1s.
   (a) Give a specification for $\mathcal{B}$.
   (b) Give an equation for the generating function of $\mathcal{B}$.

(3) Prove that the number of paths from (x, y) to $(2n, 0)$ using the steps $(1, 1)$ and $(1, -1)$ which *do* go strictly below the x-axis is the same as the number of paths from from $(x, -y-2)$ to $(2n, 0)$ using the same steps.

(4) Give the Prüfer code associated to the following tree



(5) Give a recursive generator for the class

$$\mathcal{C} = \mathcal{Z} + \mathcal{Z} \times \mathcal{C}^3$$

You may assume that you are given the counting sequence $c_0, c_1, \ldots$.

(6) Suppose that $\mathcal{A} = \mathcal{B} \times \mathcal{C}$. Suppose you have a Boltzmann generator for $\mathcal{B}$ and $\mathcal{C}$. Give pseudocode for a Boltzmann generator for $\mathcal{A}$.

(7) Suppose that $\mathcal{A} = \mathcal{B} \times \mathcal{C}$. Prove that using the Boltzmann probability for objects of $\mathcal{A}$, the first and second coordinates are independent.

(8) Here is the successor algorithm for the reflected binary Gray code. Prove that it works.

```
input: n, w.   w is a binary word of length n

result = w
if d(w) is even
   flip the last bit of result
else
   j=n
   while result(j) = 0 and j > 0
      j = j-1
   if j=1
      return no successor
   flip (j-1)th bit of result

return result
```

(9) This question concerns the revolving door order on k-subsets which we discussed in class. Prove that for all $1 \leq k \leq n$ the first k-subset of $\{1, \ldots, n\}$ in this order is

$$\{1, \ldots, k\}$$

and the last one is

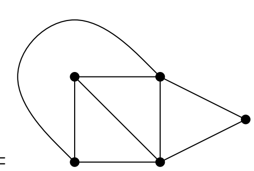
$$\{1, \ldots, k-1, n\}$$

(10) What is the rank of the permutation $[n, n-1, \ldots, 1]$ in the Trotter-Johnson order on permutations?

(11) We say a permutation σ has a 132 pattern if there are indices $i < j < k$ with $\sigma(i) < \sigma(k) < \sigma(j)$.
  (a) Give the generating tree for permutations with no 132 pattern down to permutations of $\{1, 2, 3, 4\}$.
  (b) Give the rule for the generating tree for permutations with no 132 pattern and justify that the rule is correct.

(12) Write pseudocode for a backtracking algorithm to solve the Knapsack problem.

(13) Let

$G =$ 

Find all cliques of G using a backtracking algorithm. Show the steps of the algorithm.

(14) Write pseudocode for a simulated annealing algorithm to find a largest maximal clique of a graph.

(15) Given a partial solution $X = [x_0, \ldots, x_{l-1}]$ to the Knapsack problem, define

$$B(X) = \sum_{i=0}^{l-1} p_i x_i + \texttt{RationalKnapsack}(p_l, p_{l+1}, \ldots, p_{n-1}, w_l, w_{l+1}, \ldots, w_{n-1}, M - \sum_{i=0}^{l-1} w_i x_i)$$

where `RationalKnapsack` gives the optimal profit to the rational Knapsack problem with the given parameters. Prove that $B(X)$ is a bounding function for the usual Knapsack problem.