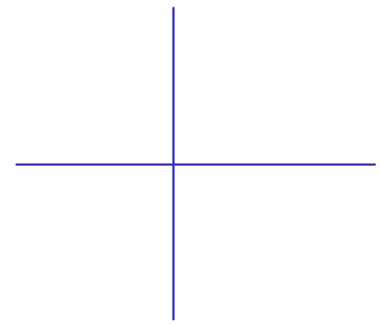
Math 303, Fall 2011, Lecture 4

1. Ordered pairs



What do we need our ordered pair to do?
what properties should it have

The standard answer is

$(a, b) =$

Note

Halmos says:

It is easy to locate the source of the mistrust and suspicion that many mathematicians feel toward the explicit definition of ordered pair given above. The trouble is not that there is anything wrong or anything missing; the relevant properties of the concept we have defined are all correct (that is, in accord with the demands of intuition) and all the correct properties are present. The trouble is that the concept has some irrelevant properties that are accidental and distracting. The theorem that $(a, b) = (x, y)$ if and only if $a = x$ and $b = y$ is the sort of thing we expect to learn about ordered pairs. The fact that $\{a, b\} \in (a, b)$, on the other hand, seems accidental; it is a freak property of the definition rather than an intrinsic property of the concept.

This is very typical of the things we will build in set theory

Now lets check our definition really does capture what it should mean to be an ordered pair.

That is, we want that if $(a,b) = (x,y)$

then $a = x$ and $b = y$

So suppose $(a,b) = (x,y)$

if (a,b) has one element

if (a,b) has 2 elements

How do we pull out the first and second coordinates?

eg $(1, 2) =$

eg $(\{\emptyset\}, \emptyset) =$

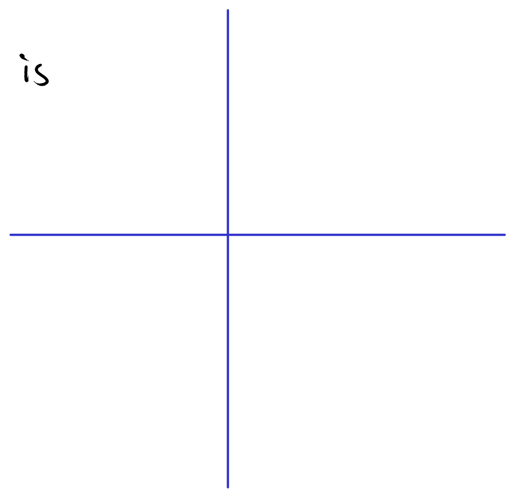
eg what ordered pair is $\{\{c, 18\}, \{18\}\}$?

eg what ordered pair is $\{\{1, \{2, \{3\}\}\}, \{\{2, \{3\}\}\}\}$

How do we build ordered pairs just with our axioms so far.

2 Cartesian products

The cartesian plane is



our good old coordinate plane.

It is

In general

Yes

Suppose $a \in A$ and $b \in B$

Now use specification to cut it down to just ordered pairs

Define the cartesian product of A and B to be

$$A \times B = \{ x \in \mathcal{P}(\mathcal{P}(A \cup B)) : x = (a,b) \text{ for some } a \in A \text{ and some } b \in B \}$$

eg

eg $A = \{1\}$, $B = \{2,3\}$

$A \times B =$

$B \times A =$

eg $A = \emptyset$ $B = \{1,2,3\}$

$A \times B = ?$

(3) Next time

A similarly artificial, but does what we need definition of numbers

Please read Halmos sections 11, 12, and 13