WHAT TO THINK ABOUT FOR CHAPTERS 5-7

While I'm away I hope you will read Chapter 5 of Spivak and take a look at chapters 6 and 7 (as described below). I've noted some problems to think about. Please meet at the usual time and place and discuss any sticky spots in the exposition and the problems. I know everyone's busy, but remember you are doing this instead of having someone prepare lectures for these two weeks, so please each spend at least the time you would have spent preparing half a lecture in reading and thinking. **What I concretely want from you is indicated in bold.**

CHAPTER 5

There are two main things to get out of Chapter 5

(1) A point p and a vector field X in a neighbourhood of p is the same information as an initial condition and a differential equation. Understand how to translate between these languages. The main result is local existence and uniqueness (Theorem 2). Think of the solution as flowing out from p according to X. Get a consistent family of flows in a neighbourhood of p.

(2) Use the above to define the Lie derivative of
  (a) a function with respect to a vector field
  (b) a covariant vector field with respect to a vector field
  (c) a vector field with respect to a vector field
  Key facts
  (a) $L_X(Y) = XY - YX = [X, Y]$ and so the Lie bracket identities are satisfied.
  (b) $[fX, gY] = fg[X, Y] + f(Xg)Y - g(Yf)X$
  (c) $[X, Y] = 0$ iff there is a coordinate system so that $X = \frac{\partial}{\partial x^1}$, $Y = \frac{\partial}{\partial x^2}$, and analogously for more than two vector fields (Theorem 14).

Problems to think about: 5-8, 5-10, 5-11, 5-13, 5-15. **Discuss which problem you would like to hand in. Email me a summary of the discussion and a conclusion if you come to one.**

CHAPTER 6

Chapter 6 is not to the point for us. The one thing to get out of it, for anyone who is going to talk to or become a practising differential geometer, is what is meant by an integrability condition (see p189) and a foliation (see p194). Glance at these so if you meet them out in the world of mathematics you know what you are ignorant about and where to it look up.

CHAPTER 7

This chapter is a study of differential forms in three steps

(1) define $\Omega^k(V)$ and understand it algebraically
(2) define k-forms as sections of $\Omega^k(TM)$ and understand how these interact with d and the Lie bracket of vector fields.

(3) define and understand closed and exact.

Almost everything in this chapter is important, but fortunately most of it is also quite easy. **Conclude amongst yourselves how much of this chapter you understand and decide where you want to pick up on March 22. Email me your conclusion and any relevant factors or caveats.**

Problems to think about (if you get that far) 7-5, 7-6, 7-7, 7-8, 7-9, (if you worked on 4-10 look at 7-12), 7-14, 7-17, 7-18, 7-21, 7-27.