ASSIGNMENT 6

MATH 303, FALL 2011

Instructions: *Do at least **3 points from each section** and at least **10 points total**. Up to 12 points will be graded, but your maximum score is 10. If you hand in more than 12 points please indicate which ones you want graded, otherwise the first 12 will be graded.*

Manipulation

(M1) **(1 point)** Let c and c' be constant symbols. Show that $\exists x\big((c = x) \wedge (x = c')\big) \to c = c'$ is valid using Cohen's rules (state explicitly which rules you use).

(M2) **(2 points)** Let $A(x)$ be a well formed formula with free variable x and let B be a well formed formula with no occurence of x. Let c be a constant symbol and let

$$S = \{\forall x A(x), A(c) \to B\}$$

Show that you can derive B from S. Use *only* Cohen's rules, and state explicitly which rules you use in your derivation.

(M3) **(1 point)** Let $E = \{a, b, c, d\}$. Let $X = \mathcal{P}(E) - \{\{a, b\}, \emptyset\}$. What are the maximal elements of X? What are the minimal elements of X? Does X have a smallest element? Does X have a largest element?

(M4) **(1 point)** Explicitly describe the order in $((\omega^+)^+)^+$.

(M5) **(1 point)** Show that $\omega 3$ is the same size as ω.

Pure Math

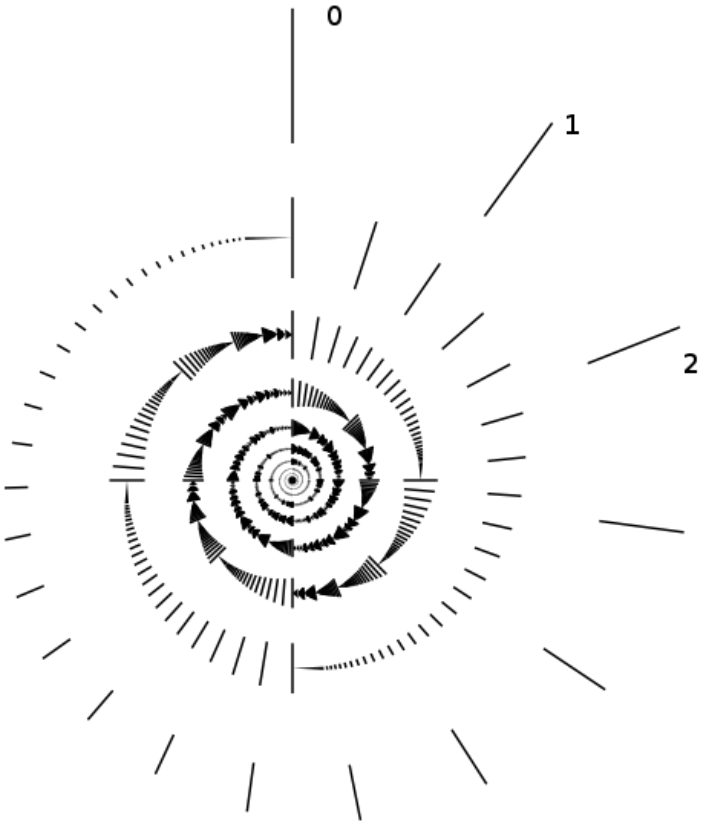
(P1) **(3 points)** Let X be partially ordered by $\leq$. Prove the following
- (a) If a is a least element of X then a is also a minimal element of X.
- (b) If X has a least element then it has a unique least element.

(P2) **(3 points)** Let $S = \left\{\forall x \forall y \forall z \left(\big(\forall w (R(x, y, w) \leftrightarrow R(x, z, w))\big) \to (y = z) \right)\right\}$.
- (a) Find an interpretation for R (taking 3 inputs) in $\mathbb{Z}$ so that $\mathbb{Z}$ is a model for S.
- (b) Find an interpretation for R (taking 3 inputs) in $\mathbb{Z}$ so that $\mathbb{Z}$ is not a model for S.

(P3) **(3 points)** Show that a countable union of countable sets is countable.

Ideas

(I1) **(3 points)** Here is an illustration of the ordinal numbers up to ω^ω.



(Source: `http://en.wikipedia.org/wiki/File:Omega-exp-omega.svg`). Label the ticks with names of the appropriate ordinals until they get too small to label. I got you started with the first 3. (If you need more hints, go see the description in the source link).

(I2) **(3 points)** Compare the Burali-Forti paradox to Russell's paradox. Mention both the structure of the paradoxes themselves and how we resolved them.