

Why are some two-step Runge–Kutta methods inaccurate?

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Abstract. Jackiewicz and Verner [2] derived and tested the implementation of formulas for explicit two-step Runge–Kutta (TSRK) pairs. Although methods eventually constructed satisfied the order conditions, simulations on several problems indicated that the achieved order for a method of order eight was lower than expected.

More recent experiments have illustrated that two-step Runge–Kutta methods with stage order at least 2 less than the propagation order may not achieve the design order. The error introduced at the outset and its propagated values have a predictable behaviour, and so it was expected that changes in the implementation might rectify the deficiency.

Detailed analysis of formulas used to derive TSRK methods indicated that improvements in the achieved order might be possible using a strategy developed by Butcher [1] for implementing conventional Runge–Kutta methods of "effective" order. Essentially, the starting values have to assume a pattern which is designed to be propagated by the TSRK formula. Appropriate "starting" methods were designed and constructed for TSRK methods of orders 4 and 6. Formulas used in deriving these starting methods, and evidence that the design order is achieved will be presented. Hence, production codes are expected to improve on some results in [2].

References

- [1] J.C. Butcher, *The effective order of Runge–Kutta methods*, Conf. on the Numerical Solution of Differential Equations, Dundee, 1969, Lecture Notes in Mathematics No. 109, Springer, Berlin, p. 133-139.
- [2] Z. Jackiewicz and J.H. Verner, *Derivation and implementation of two-step Runge–Kutta pairs*, Japan JIAM 19 (2002), pp. 227–248.

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