A Retrospective on the Derivation of Explicit Runge—Kutta Pairs*

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Abstract

To improve the efficiency of estimating solutions to initial value problems in ordinary differential equations C. Runge and W. Kutta designed a recursive evaluation approach to obtaining estimates within a {\it step} that led to a high-order polynomial approximation (1895-1905). As Kutta had difficulty with obtaining correct coefficients of a method of order 5, a mathematical approach to tabulating and solving the polynomial order conditions evolved, primarily after 1957. Although Butcher's proposal of 'simplifying conditions' led to new families of methods, solution of the order conditions remained a challenge. The proposal to provide a pair of RK approximations to control the step-by-step error increased this complexity. Yet, Butcher's "Algebraic Theory of Integration Methods" and new pairs up to orders 8,9 found by Fehlberg motivated new approaches, and derivation of better Runge—Kutta pairs. This talk will survey some of the strategies used since 1970 to find new formulas, and will suggest how these tools may be applied to finding algorithms of different structures or for different problems.