Numerically Optimal Runge–Kutta Pairs and Interpolants

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Since Fehlberg [1] derived pairs of Runge–Kutta methods of adjacent high orders to provide an error estimate at each step, optimal pairs have been selected by attempts to minimize the 2-norm of the local truncation error coefficients of the higher order method. From a modification designed by the author [3], various types of pairs have been classified [4], and algorithms for many of these types have been developed (See [5]). In attempts to reduce the LTE 2-norm, modifications and different types of pairs of methods have been developed by other authors (e.g. [2]). This presentation suggests other attractive features of a pair that can and should be considered in a selection of optimal pairs. The author's algorithms are used to obtain pairs of order p and p-1, p = 6,7,8,9, with interpolants of the same orders that are 'numerically optimal' (LTE 2-norm is small), and with some compromise, some formulas have additional attractive features. For these, the LTE 2-norms are lower than those of previously published formulas, numerically optimal interpolants are provided, and all coefficients can be obtained exactly. (In practice, 40 digit approximations are provided at (www.math.sfu.ca/~jverner), and more accurate coefficients can be provided to interested researchers by email on request.)

While coefficients on the author's website provide 'most efficient' and 'most robust' algorithms which might be used to improve the efficiency of existing software for initial value problems, one of the principal motivations for this project was to obtain the 'most accurate' algorithms possible which may be applied to standard test problems to order to calibrate corresponding test results obtained from two-step Runge–Kutta methods now under development.

References

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