Improved Starting Methods for Two-step Runge–Kutta Methods of Low Stage-Order

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Abstract.

Explicit Runge-Kutta pairs of orders 5 and 6 require 8 derivative evaluations per step. An extension of two-step Runge–Kutta (TSRK) methods designed by Jackiewicz and Tracogna [2] led to TSRK pairs of corresponding orders [3] requiring only 4 new stages per step. Implementation with stepsize change requires interpolation, and although the total computation is only slightly more than for RK pairs, the expected design order was not achieved. In experiments with Philip Sharp using fixed stepsizes, the deficiency was identified as a necessity that initial off-step derivative values be "perturbed" to match the error profile of off-step values in subsequent steps. Hairer and Wanner [1] first identified this requirement for TSRK methods.

In the talk, initial deficiencies will be clearly illustrated. The first "starting" methods derived depended on a rather bizarre polynomial condition. An improved design yields lower-stage starting methods with a more standard derivation. Tabulated results from starting methods implemented with fixed stepsizes will illustrate that the desired order can be achieved. Still, the discovery of an efficent, correct implementation with changes in stepsize of these *low stage-order* TSRK methods remains a challenge.

References

[1] E. Hairer and G. Wanner, *Order conditions for general two-step Runge–Kutta methods.*, SIAM J. Numer. Anal. **34** (1997), pp. 2087-2089.

[2] Z. Jackiewicz and S. Tracogna, *A general class of two-step Runge–Kutta methods for ordinary differential equations*, SIAM J. Numer. Anal. **32** (1995), pp. 347–368.

[3] Z. Jackiewicz and J.H. Verner, *Derivation and implementation of two-step Runge–Kutta pairs*, Japan JIAM **19** (2002), pp. 227–248.

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