# Coast Combinatorics Conference 2023

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## Abstracts (in alphabetical order of family name)

**Ben Cameron**, The King's University
*Vertex-critical ($P_3 + \ell P_1$)-free graphs*

A graph $G$ is $k$-vertex-critical if $\chi(G) = k$ but $\chi(G-v) < k$ for all $v \in V(G)$ where $\chi(G)$ denotes the chromatic number of $G$. Due to the direct connection to polynomial-time certifying algorithms for deciding $k$-colourability, there has been considerable recent interest in classifying $k$-vertex-critical graphs in various graph families. In this talk, we will discuss why $k$-vertex-critical $(P_3 + \ell P_1)$-free graphs are of particular interest and sketch the details of our proof that there are only finitely many such graphs for all $k$ and $\ell$. Together with previous results, the only graphs $H$ for which it is unknown if there are an infinite number of $k$-vertex-critical $H$-free graphs is $H = (P_4 + \ell P_1)$ for all $\ell \geq 1$. We will also discuss our progress on these outstanding cases by giving our characterization for $k$-vertex-critical (gem, co-gem)-free graphs.

Joint work with T. Abuadas (Wilfrid Laurier University), C. T. Hoàng (Wilfrid Laurier University), and J. Sawada (University of Guelph).

**MacKenzie Carr**, Simon Fraser University
*Improper interval colorings of outerplanar graphs and corona products*

A proper edge coloring of a graph $G$ is an interval $k$-coloring if exactly $k$ colors are used and the colors of the edges incident to every vertex $v \in V(G)$ are distinct and form a set of consecutive integers. There exist graphs that do not have an interval $k$-coloring, for any value of $k$. In 2021, Casselgren and Petrosyan studied improper interval colorings, in which the colors of the edges incident to each vertex form an integral interval, but are not necessarily distinct. An improper interval coloring is $k$-improper if at most $k$ edges with the same color all share a common endpoint, and the smallest integer $k$ such that $G$ has a $k$-improper interval coloring is called the interval coloring impropriety of $G$. In this talk, we show that the interval coloring impropriety of an outerplanar graph $G$ with $\Delta(G) \geq 6$ is at most $\lceil \frac{\Delta(G)}{5} \rceil$. Moreover, we determine upper bounds on the interval coloring impropriety of the corona products of several classes of graphs, including paths, cycles and caterpillars.

Joint work with Eun-Kyung Cho (Hankuk University of Foreign Studies), Nicholas Crawford (University of Colorado Denver), Vesna Iršič (University of Ljubljana/Institute of Mathematics, Physics and Mechanics), Leilani Pai (University of Nebraska - Lincoln), Rebecca Robinson (University of Colorado Denver).

**Alexander Clow**, Simon Fraser University
*Oriented & 2-dipath colouring*

This talk focuses on the relationship between the oriented chromatic number ($\chi_o$) and the 2-dipath chromatic number ($\chi_2$). In particular we improve an upper bound of MacGillivray, Raspaud, and Swarts of the form $\chi_o \leq 2^{\chi_2} - 1$ to $\chi_o \leq \chi_2 f(d) 2^d$ where $\chi_2 \leq 2^{\frac{f(d)}{d}}$, $f(d) = \omega 2^d$ such that $\omega \to \infty$ with $d$. Here $d$ is the degeneracy of the graph in question.

Joint work with Peter Bradshaw (University of Illinois Urbana Champaign) and Ladislav Stacho (Simon Fraser University, Burnaby).

**James D. Currie**, University of Winnipeg
*A characterization of the period-doubling word*

The *Thue-Morse sequence* $\mathbf{t}$ is the fixed point of the map $h : 0 \mapsto 01$, $1 \mapsto 10$. In 1906, Thue proved that for any sequence avoiding *overlaps*, a final segment is the image under $h$ of an overlap-avoiding sequence. Many published results involving $\mathbf{t}$ result from this structure theorem.

Another basic sequence in combinatorics on words is the *period doubling sequence* $p$, the fixed point of the map $g : 0 \mapsto 01$, $1 \mapsto 00$. We show for any *good* sequence — i.e., one avoiding certain patterns and factors — a final segment is the image under $g$ of a good sequence. One anticipates that analogues of the (many) results proved for $\mathbf{t}$ should therefore exist for $p$.

**Ryan B. Hayward**, University of Alberta
*Notes on Dark Hex*

*Dark Hex* — also called Kriegspiel Hex or Phantom Hex — is an imperfect-information version of Hex. Dark Hex can be played on any $m \times n$ Hex board with $m$ rows and $n$ columns. Each player knows only their own moves: they do not see their opponent's moves. The umpire sees all moves. For $m \times n$ Dark Hex with $m$ at least 3 and $n$ at least 4, neither player has a deterministic strategy that always wins, so we seek a minimax solution: for each player, we want a weighted mixed strategy that maximizes their expected winrate over all possible deterministic opponent strategies. For $3 \times 4$ or larger-board Dark Hex the number of possible strategies is so large that finding an exact solution with the standard methods is intractable, so we seek solution bounds. For $3 \times 4$ Dark Hex, François Bonnet found that the minimax expected winrate $p$ of the first-player (who we assume owns the edges that are distance 4 apart) satisfies $.112 \leq p \leq .268$. We give simple strategies that improve these bounds: $1/7 = .142 \ldots \leq p \leq .25$.

Joint work with Martin Müller and Bedir Tapkan.

**Brian Hopkins**, Saint Peter's University
*The mex of an integer partition*

A young Freeman Dyson discovered the rank of a partition, a simple statistic which provided combinatorial perspectives on some congruences found by Ramanujan. However, one congruence needed something else that he could not find, but named anyway: the crank. It was finally found more than 40 years later by George Andrews and Frank Garvan. The definition of the crank is a bit tricky, but it has gone on to become a major topic in the study of integer partitions.

A more recent statistic is the minimal excluded part of a partition, known as the mex, borrowing a term from combinatorial game theory. This simpler parameter has surprising connections to the crank and is more amenable to combinatorial interpretations. The mex of integer partitions has become a popular topic recently, discussed in at least a dozen papers within the last three years and with more on the way.

In this survey, I will mention joint work with James Sellers (University of Minnesota Duluth), Dennis Stanton (University of Minnesota), and Ae Ja Yee (Pennsylvania State University).

**Joseph Hyde**, University of Victoria
*The Kohayakawa-Kreuter conjecture holds for almost all pairs of graphs*

We study asymmetric Ramsey properties of the random graph $G_{n,p}$. For $r \geq 2$ and a graph $H$, Rödl and Ruciński (1993-1995) provided the asymptotic threshold for $G_{n,p}$ to have the following property: whenever we $r$-colour the edges of $G_{n,p}$ there exists a monochromatic copy of $H$ as a subgraph. In 1997, Kohayakawa and Kreuter conjectured an asymmetric version of this result where one replaces $H$ with a set of graphs $H_1, \ldots, H_r$ and we seek the threshold for when every $r$-colouring contains a monochromatic copy of $H_i$ in colour $i$ for some $i \in \{1, \ldots, r\}$. The 1-statement of this conjecture was confirmed by Mousset, Nenadov and Samotij in 2020.

Building on work of Hyde (2022), we resolve the 0-statement of Kohayakawa and Kreuter's conjecture for almost all sequences of graphs $H_1, \ldots, H_r$. The remaining cases we reduce to a deterministic colouring problem.

Joint work with Candida Bowtell (University of Warwick) and Robert Hancock (Universität Heidelberg).

**Mark Kayll**, University of Montana
*Deranged matchings: confessions and corrections*

Godsil, in [*Algebraic Combinatorics*, 1993], remarks on derangements that "counting them is a traditional preoccupation of Combinatorics texts." The speaker, in past abstracts [2017–22], has quoted this before. So perhaps by now, we've ***really*** seen enough on derangements. However, with the paper now accepted, we feel compelled to atone for past white lies on the topic. (Joint work with Dan Johnston and Cory Palmer).

**Amirhossein Kazeminia**, Simon Fraser University
*Complexity classification of counting graph homomorphisms modulo a prime number*

Counting graph homomorphisms and its generalizations such as the Counting Constraint Satisfaction Problem (CSP), variations of the Counting CSP, and counting problems in general have been intensively studied since the pioneering work of Valiant. While the complexity of exact counting of graph homomorphisms (Dyer and Greenhill, 2000) and the Counting CSP (Bulatov, 2013, and Dyer and Richerby, 2013) is well understood, counting modulo some natural number has attracted considerable interest as well. In their 2015 paper Faben and Jerrum suggested a conjecture stating that counting homomorphisms to a fixed graph $H$ modulo a prime number is hard whenever it is hard to count exactly, unless $H$ has automorphisms of certain kind. In this paper we confirm this conjecture. As a part of this investigation we develop techniques that widen the spectrum of reductions available for modular counting and apply to the general CSP rather than being limited to graph homomorphisms.

Joint work with Andrei A.Bulatov (Simon Fraser University).

**Shuxing Li**, Simon Fraser University
*A combinatorial perspective towards the planarity of functions*

Given a function $f : \mathbb{F}_2^n \to \mathbb{F}_2^n$, the planarity of $f$ describes the correlation between different outputs of $f$. Since 1990s, planarity has become a major cryptographic criterion measuring the resistance of a function against the differential cryptanalysis. Taking a combinatorial perspective, we associate each function with a configuration named partial quadruple system, which retains fruitful information about its planarity. We relate the partial quadruple system to some well-known quantities describing the planarity, and explicitly determine the partial quadruple system in some special cases.

Joint work with Wilfried Meidl (University of Klagenfurt), Alexandr Polujan (Otto von Guericke University Magdeburg), Alexander Pott (Otto von Guericke University Magdeburg), Constanza Riera (Western Norway University of Applied Sciences), Pantelimon Stănică (Naval Postgraduate School).

**Monty McGovern**, University of Washington
*Harmonic degrees of representations of classical Weyl groups*

Stanley has shown that the degrees in which irreducible representations of the symmetric group occur in its covariant algebra are given by the major indices of the standard tableaux of the corresponding shape. I will generalize this result to hyperoctahedral groups, using domino tableaux rather than standard ones, and partially generalize it to type $D$ as well.

**Sean McGuinness**, Thompson Rivers University
*A serial base exchange property for random bases*

In a matroid, the symmetric base exchange property asserts that for any bases $A$ and $B$ and for any element $e \in A$, there is an element $f \in B$ for which $A - e + f$ and $B - f + e$ are bases. In generalizing this, it has been conjectured that for any two bases $A, B$, and any $k$-subset $A_1 \subseteq A$, there is a $k$-subset $B_1 \subseteq B$ such that one can exchange the elements of $A_1$ with the elements of $B_1$ in some order so that after each exchange, the resulting sets are bases. Viewing this problem in the vector space $F_q^n$, we ask the question, if the bases $A$ and $B$ are randomly chosen, and the $k$-subset $A_1 \subseteq A$ is randomly chosen, what is the probability that there is a $k$-subset $B_1 \subseteq B$ for which $A_1$ can be serially exchanged with $B_1$?

**Lucas Mol**, Thompson Rivers University
*Avoiding additive powers in words*

A *word* is a sequence of symbols taken from some finite alphabet. A *square* is a word of the form $xx$, where $x$ is a nonempty word. It is well-known that there are infinite words over an alphabet of size 3 that contain no squares.

Suppose now that the alphabet is some finite subset of the integers. An *additive square* is a word of the form $x\tilde{x}$, where $x$ and $\tilde{x}$ have the same nonzero length and the same sum. Additive cubes, fourth powers, etc., are defined similarly. We present a method for proving that certain types of infinite words contain no additive $k$-powers. This is joint work with James Currie, Narad Rampersad, and Jeffrey Shallit.

**Daniel Neuen**, Simon Fraser University
*The iteration number of the Weisfeiler-Leman Algorithm*

The Weisfeiler-Leman procedure is a widely-used technique for graph isomorphism testing that works by iteratively computing an isomorphism-invariant coloring of vertex tuples. More precisely, for a fixed integer $k \geq 1$, it iteratively refines a coloring of vertex $k$-tuples by aggregating local structural information encoded in the colors. In this talk, I present new upper and lower bounds on the number of iterations the $k$-dimensional Weisfeiler-Leman algorithm ($k$-WL) requires until stabilization.

For $k \geq 3$, we obtain that $k$-WL stabilizes after at most $O(kn^{k-1} \log n)$ iterations (where $n$ denotes the number of vertices of the input graph), obtaining the first improvement over the trivial upper bound of $n^k - 1$ and extending a previous upper bound of $O(n \log n)$ for $k = 2$ [Lichter et al., LICS 2019].

We complement our upper bounds by constructing $k$-ary relational structures on which $k$-WL requires at least $n^{\Omega(k)}$ iterations to stabilize.

The number of iterations required by $k$-WL to distinguish two structures corresponds to the quantifier rank of a sentence distinguishing them in the $(k+1)$-variable fragment $C_{k+1}$ of first-order logic with counting quantifiers. Hence, our results also imply new upper and lower bounds on the quantifier rank required in the logic $C_{k+1}$.

Joint work with M. Grohe (RWTH Aachen University) and M. Lichter (TU Darmstadt).

**Kathryn Nurse**, Simon Fraser University
*Nowhere-zero flows and group-connectivity in signed graphs*

In 1954, Tutte made a conjecture that every graph without a cut-edge has a nowhere-zero 5-flow. A parallel conjecture to this, but for signed graphs, is Bouchet's Conjecture (1983) that every signed graph without the obvious obstruction has a nowhere-zero 6-flow. We prove that Bouchet's Conjecture holds in the special case of 3-edge-connected graphs when 6 is replaced with 8. The notion of flows in graphs was generalized to group-connectivity by Jaeger, Linial, Payan, and Tarsi (1992); and to group-connectivity in signed graphs by Li, Luo, Ma, and Zhang (2018). We prove that if a signed graph is 3-edge-connected and 2-unbalanced, then it is $A$-connected for every abelian group $A$ when $|A| \geq 6$ and $|A| \neq 7$.

Based on joint works with A. Brewer Castano (Auburn University), M. DeVos (Simon Fraser University), J. McDonald (Auburn University), and R. Šámal (Charles University).

**Daniel Panario**, Carleton University
*Quantum QC-LDPC Codes with large girth*

We show that all quantum quasi-cyclic LDPC (QQC-LDPC) codes with column weight at least 3 have girth at most 6. We also present an efficient method to construct QQC-LDPC codes with column weight 2 and girth 12.

Joint work with Farzane Amirzade (Carleton University) and Mohammad-Reza Sadeghi (Amirkabir University of Technology).

**Gara Pruesse**, Vancouver Island University
*Greed is good for 2-approximating the jump number for interval posets*

The jump number problem for interval posets is NP-hard. The jump number is realized by a linear extension that minimizes the number of times it *jumps* from the end of one chain of elements to the start of another chain. Jump number is one of the most studied poset metrics. There are algorithms to 3/2-approximate (and slightly better) the jump number for interval posets (Felsner, Sysło, and Krysztowiak each have papers giving such algorithms); these algorithms are more complex to implement than plain greed. Plain greed refers to shelling the elements of the poset, always avoiding a jump unless there is no other option. Plain greed applied to general posets can be pretty bad: it can yield linear extensions with $n/2$ times the optimal number of jumps, that is, an approximation ratio of $n/2$. However, greed's effectiveness for approximating jump number for interval posets has been a matter of interest for decades. We show that greed achieves an approximation ratio of 2 for the jump number of interval posets; this is the first time an approximation ratio has been proved for plain greed on this class. Furthermore, the bound is tight, and we give an infinite class of interval posets for which greed yeilds exactly twice the optimal jump number.

This resolves a decades old question arising from Felsner (1990), and of Faigle & Schrader (1985), about the effectiveness of greed for the jump number of interval posets.

**Amites Sarkar**, Western Washington University
*Bootstrap percolation in random geometric graphs*

Random geometric graphs were invented by E.N. Gilbert in 1961 to model communications networks. Bootstrap percolation was invented by Chalupa, Leath and Reich in 1979 to model magnetism. Both models have since been used to study many other things. This is because many spatial networks, such as the brain, can be modeled as random geometric graphs, and many natural processes on networks, such as the activation of neurons in the brain, or the spread of beliefs on social media, can be modeled by bootstrap percolation. So it was natural when, in 2014, Bradonjic and Saniee put the two models together, and studied bootstrap percolation in random geometric graphs. In my talk, I'll describe an (almost) complete solution of the Bradonjic-Saniee model. The proofs use variational methods, tiling arguments, and discrete isoperimetric inequalities.

Joint work with Victor Falgas-Ravry (Umeå University).

**Chi Hoi Yip**, University of British Columbia
*Intersective sets over abelian groups*

Given a finite abelian group $G$ and a subset $J \subset G$ with $0 \in J$, let $D_G(J, N)$ be the maximum size of $A \subset G^N$ such that the difference set $A - A$ and $J^N$ have no non-trivial intersection. Recently, this extremal problem has been widely studied for different groups $G$ and subsets $J$. In this talk, I will present a connection between this problem and cyclotomic polynomials with the help of algebraic graph theory. In particular, we construct infinitely many non-trivial families of $G$ and $J$ for which the current known upper bounds on $D_G(J, N)$ can be improved exponentially.

Joint work with Zixiang Xu (Institute for Basic Science, South Korea).

**Hanmeng (Harmony) Zhan**, Simon Fraser University
*Discrete quantum walk search on graphs*

Discrete quantum walks are motivated by search problems. One of the best known quantum algorithms, Grover's search, is a discrete quantum walk on the complete graph with loops. In 2003, Shenvi, Kempe and Whaley proposed a quantum walk algorithm that finds marked vertices on a general graph. Since then, their algorithm has been studied on various graphs, including hypercubes, Cartesian powers of cycles, strongly regular graphs and Johnson graphs. For most of these graphs, quantum walks find the target faster than classical random walks.

In this talk, we consider the search problem from an averaging perspective: given a graph and a marked vertex, what is the average probability, over any period of time, that a quantum walk finds the marked vertex? We show that for any family of distance regular graphs meeting a valency-size criterion, this average success probability approaches 1/4, regardless of the defining parameters. This is a consequence of many neat results in algebraic graph theory, group theory and orthogonal polynomials.