Math 251, Calculus III

J. Hebron, Spring 2000

# Mid-Term Examination #2

Friday, March 10th, 2000

Time: 55 minutes

Student ID Number

Name
(Please underline your family name)

Signature

**Instructions:**

- **Please fill-in the above information in ink.**
- **Do not open this exam until told to do so.**
- **No books, no notes, no calculators, no cell phones.**
- **Please sign the bottom of every page**
  (in case your exam becomes unstapled)

| Question #: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Tot |
|---|---|---|---|---|---|---|---|---|---|
| Mark: | | | | | | | | | |
| Out of: | 6 | 6 | 2 | 6 | 10 | 8 | 8 | 4 | 50 |

**[Mark]**

**1.** Show $\lim_{(x,y)\to(0,0)} \frac{3y^2 - 5x^2 - 14xy}{3y^2 - 5x^2}$ doesn't exist.

**[ 6 ]**

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**2.** Find the equation for the tangent plane to the surface $ze^{x+y} - \cos(xyz) = 0$ at the point (0,0,1).

**[ 6 ]**

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**3.** Assume that the equation $ze^{x+y} - \cos(xyz) = 0$ (from problem 2) implicitly defines $z(x,y)$, where $x$ and $y$ are taken to be the independent variables and $z$ the dependent variable. What are $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the point (0,0,1)?

**[ 2 ]**

**4.** Let $f(x,y) = \ln(1+|x|) + \ln(1+|y|)$. Find and classify all critical points.

**[ 6 ]**

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**5.** Let $f(x,y) = \left(x^2 - 2x\right)\left(y - 2\right)$. Find and classify all critical points.

**[ 10 ]**

(Signature)

**6.** Using the method of **Lagrange Multipliers** (*no marks will be given for any other method*), find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, where $a$, $b$, and $c$ are positive numbers.

**[ 8 ]**

(Signature)

**7.** Evaluate $\int_0^2 \int_{x/2}^1 \sin\left(y^2\right) dy dx$.

**[ 8 ]**

(Signature)

**8.** Let **D** be the quarter-annulus defined in polar coordinates by $\mathbf{D} = \left\{(r,\theta) | 1 \le r \le 2, 0 \le \theta \le \frac{\pi}{2}\right\}$. Find $\iint_{\mathbf{D}} \frac{1}{\sqrt{x^2+y^2}} dA$.

**[ 4 ]**

(Signature)