Math 251, Calculus III

J. Hebron, Fall 1999

Mid-Term Examination #1

Wednesday, Oct 6th, 1999

Marks

Total mark out of [100]

1. Consider the space curve defined by the following:

$$x = e^t \cos t$$
$$y = e^t \sin t$$
$$z = 0$$

and assume there is a particle moving along this curve as a function of time t.

(a) Sketch the curve. [2]
(b) Find the velocity vector and its magnitude. [4]
(c) Find the acceleration vector and its magnitude [4]
(d) What is the unit tangent vector $\vec{\mathbf{T}}$? [2]
(e) What is the unit normal vector $\vec{\mathbf{N}}$? [4]
(f) What is the curvature of the curve? [2]
(g) What is the magnitude of the tangential component of acceleration vector? [3]
(h) What is the magnitude of the normal component of acceleration vector? [3]
(i) What is the arc length from t=0 to t=1? [3]

2. Find an equation of the plane that passes through the point (1,6,-4) and contains the line described by $x = 1 + 2t$, $y = 2 - 3t$, $z = 3 - t$. [20]

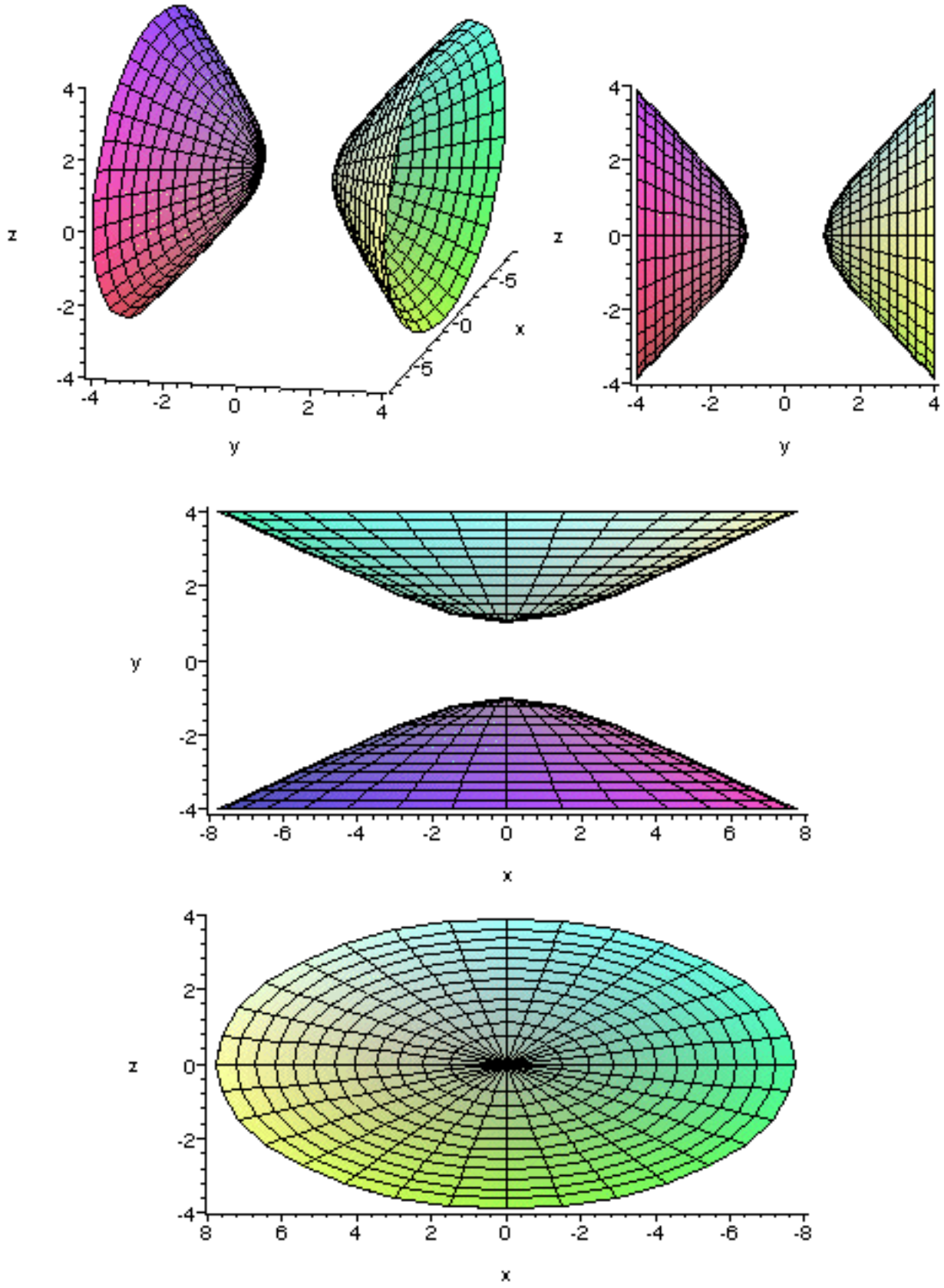
3. What is the vector identity for $\vec{\mathbf{A}} \times (\vec{\mathbf{B}} \times \vec{\mathbf{C}})$? Verify this identity for $\vec{\mathbf{A}} = \langle -4,0,3 \rangle$, $\vec{\mathbf{B}} = \langle 2,-1,0 \rangle$, $\vec{\mathbf{C}} = \langle 0,2,5 \rangle$. [20]

4. What is the equation of a sphere in cylindrical coordinates? [5]

5. What is the equation of a cylinder in spherical coordinates? [8]

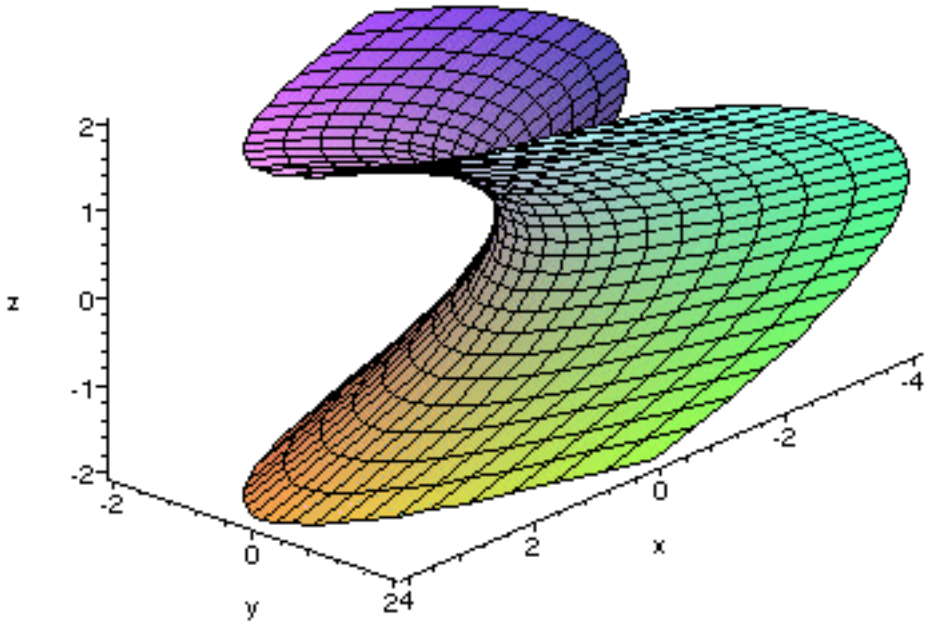
6. What is the volume of a parallelepiped with adjacent edges PQ, PR, and PS, where $P = (1,1,1)$, $Q = (2,0,3)$, $R = (4,1,7)$, and $S = (3,-1,-2)$? [10]

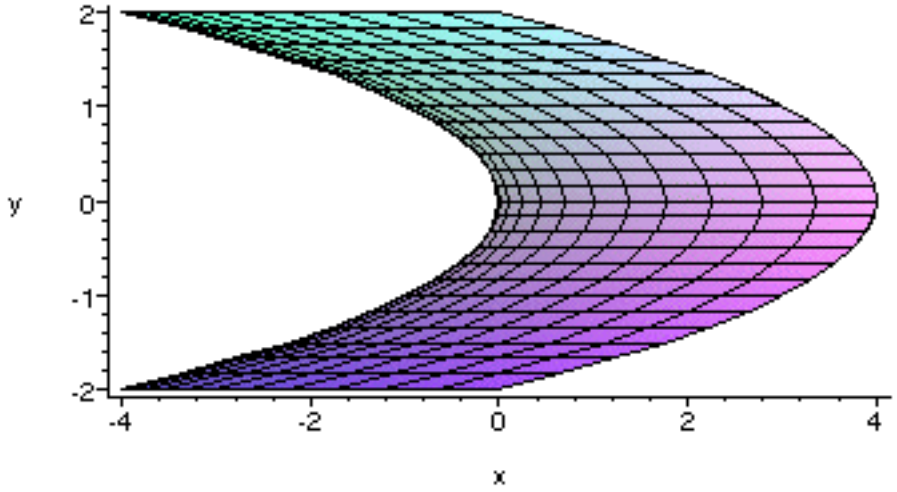
7. Consider the following surface, as shown from various views:

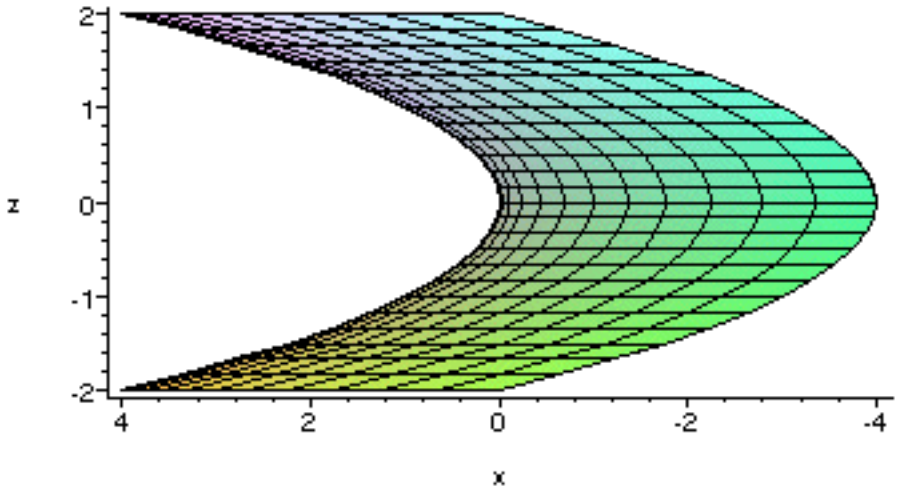


What is it's equation? [5]

8. Consider the following surface, as shown from various views:







What is it's equation? [5]