Math 251, Calculus III

J. Hebron, Spring 2000

Mid-Term Examination #1

Friday, February 4th, 2000

Time: 50 minutes

	SOLUTIONS
Student ID Number	Name (Please underline your family name)
	J.H.
	Signature

Instructions:

- **Please fill-in the above information in ink.**
- **Do not open this exam until told to do so.**
- **No books, no notes, no calculators**
- **Please sign the bottom of every page**
  (in case your exam becomes unstapled)

Question #:	1	2	3	4	5	6	7	Tot
Mark:								
Out of:	2	10	10	5	10	5	18	60

Part A: Multiple Choice

Instructions: Circle the correct answer for each question. You may use the back pages of the exam for any rough work. Note that rough work will not be marked. Marks are only awarded for the correct answer.

[mark]

1. Which vector is parallel to the line given by $\dfrac{x-a}{\alpha}=\dfrac{y-b}{\beta}=\dfrac{z-c}{\gamma}$? [2]

(a) $\langle a,b,c\rangle$ (b) $\langle \alpha,\beta,\gamma\rangle$ **(c)** $\left\langle \frac{1}{a},\frac{1}{b},\frac{1}{c}\right\rangle$ **(d)** $\left\langle \frac{1}{\alpha},\frac{1}{\beta},\frac{1}{\gamma}\right\rangle$

(e) $\left\langle \frac{\alpha}{a},\frac{\beta}{b},\frac{\gamma}{c}\right\rangle$ **(f)** $\left\langle \frac{a}{\alpha},\frac{b}{\beta},\frac{c}{\gamma}\right\rangle$ **(g)** none of the above

2. Which vector is parallel to the line of intersection of the planes $x-z=1$ and $y+z=1$? [10]

(a) $\langle 1,0,0\rangle$ **(b)** $\langle 0,1,0\rangle$ **(c)** $\langle 0,0,1\rangle$ **(d)** $\langle 1,1,0\rangle$ **(e)** $\langle 1,0,1\rangle$
(f) $\langle 0,1,1\rangle$ **(g)** $\langle 1,-1,0\rangle$ **(h)** $\langle 1,0,-1\rangle$ **(i)** $\langle 0,1,-1\rangle$ **(j)** $\langle 1,1,1\rangle$
(k) $\langle -1,1,1\rangle$ (l) $\langle 1,-1,1\rangle$ **(m)** $\langle 1,1,-1\rangle$
(n) the planes don't intersect **(o)** none of the above

3. Indicate whether the following statements are true or false.

(i)	$\vec{\mathbf{a}}\bullet\vec{\mathbf{b}}=\vec{\mathbf{b}}\bullet\vec{\mathbf{a}}$	(a) true	**(b)** false	[1]
(ii)	$\vec{\mathbf{a}}\times\vec{\mathbf{b}}=\vec{\mathbf{b}}\times\vec{\mathbf{a}}$	**(a)** true	(b) false	[1]
(iii)	$\vec{\mathbf{a}}\bullet\left(\vec{\mathbf{b}}\times\vec{\mathbf{c}}\right)=\left(\vec{\mathbf{a}}\bullet\vec{\mathbf{b}}\right)\times\vec{\mathbf{c}}$	**(a)** true	(b) false	[1]
(iv)	$\vec{\mathbf{a}}\bullet\left(\vec{\mathbf{b}}\times\vec{\mathbf{c}}\right)=\left(\vec{\mathbf{a}}\times\vec{\mathbf{b}}\right)\bullet\vec{\mathbf{c}}$	(a) true	**(b)** false	[1]
(v)	$\vec{\mathbf{a}}\times\left(\vec{\mathbf{b}}\times\vec{\mathbf{c}}\right)=\left(\vec{\mathbf{a}}\times\vec{\mathbf{b}}\right)\times\vec{\mathbf{c}}$	**(a)** true	(b) false	[1]
(vi)	$\vec{\mathbf{a}}\times\left(\vec{\mathbf{b}}\times\vec{\mathbf{c}}\right)=(\vec{\mathbf{a}}\bullet\vec{\mathbf{c}})\vec{\mathbf{b}}-\left(\vec{\mathbf{a}}\bullet\vec{\mathbf{b}}\right)\vec{\mathbf{c}}$	(a) true	**(b)** false	[1]
(vii)	$\hat{\mathbf{i}}\times\hat{\mathbf{k}}=\hat{\mathbf{j}}$	**(a)** true	(b) false	[1]
(viii)	$\dfrac{d}{dt}[\vec{\mathbf{u}}(t)\times\vec{\mathbf{v}}(t)]=\vec{\mathbf{u}}'(t)\times\vec{\mathbf{v}}(t)-\vec{\mathbf{v}}'(t)\times\vec{\mathbf{u}}(t)$	(a) true	**(b)** false	[1]
(ix)	$\dfrac{d}{dt}[f(t)\vec{\mathbf{u}}(t)]=f'(t)\vec{\mathbf{u}}'(t)$	**(a)** true	(b) false	[1]
(x)	$\hat{\mathbf{n}}(t)\bullet\hat{\mathbf{n}}'(t)=0$ (where $\hat{\mathbf{n}}(t)$ is any unit vector)	(a) true	**(b)** false	[1]

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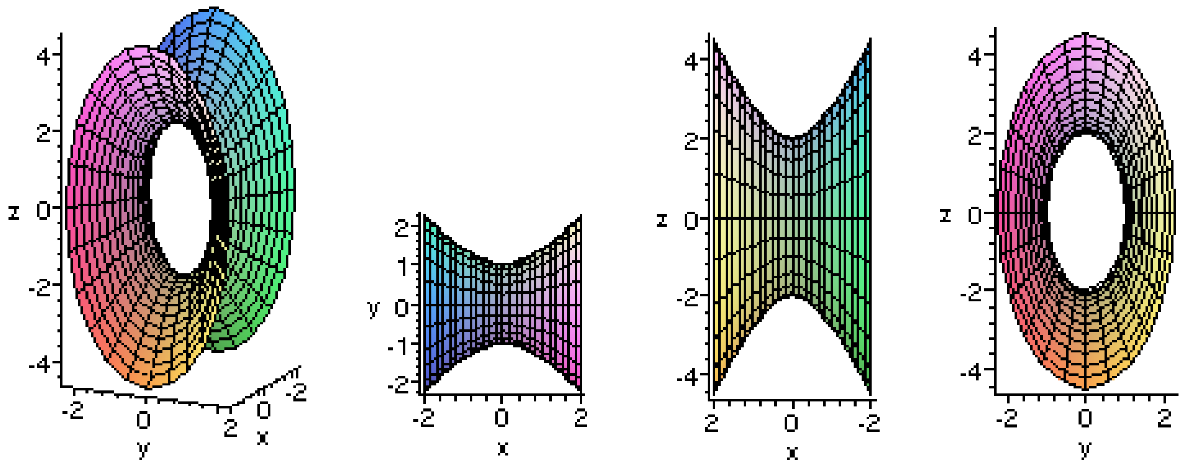
4. The following surface was plotted in Maple. It is an infinite surface, but it was cut off at $x = \pm 2$. The plot is shown from various views, as seen in Maple as the 3-d plot is rotated.



3-d View **View from z-axis** **View from y-axis** **View from x-axis**

What is the equation for this surface? **[5]**

(a) $x^2 + y^2 + \frac{z^2}{4} = 1$ **(b)** $-x^2 + y^2 + \frac{z^2}{4} = 1$ **(c)** $x^2 - y^2 + \frac{z^2}{4} = 1$

(d) $x^2 + y^2 - \frac{z^2}{4} = 1$ **(e)** $x^2 - y^2 - \frac{z^2}{4} = 1$ **(f)** $-x^2 + y^2 - \frac{z^2}{4} = 1$

(g) $-x^2 - y^2 + \frac{z^2}{4} = 1$ **(h)** $-x^2 - y^2 - \frac{z^2}{4} = 1$ **(i)** $x = y^2 + \frac{z^2}{4}$

(j) $y = x^2 + \frac{z^2}{4}$ **(k)** $x = y^2 - \frac{z^2}{4}$ **(l)** $y = x^2 - \frac{z^2}{4}$

(m) $x = -y^2 + \frac{z^2}{4}$ **(n)** $y = -x^2 + \frac{z^2}{4}$ **(o)** none of the above

5. Consider the point in space given by the cartesian coordinates $(\sqrt{3}, 1, -2)$.

(i) If this point was expressed in cylindrical coordinates, what would r be? **[2]**

(a) 1 **(b)** $\sqrt{3}$ (c) 2 **(d)** $2\sqrt{2}$ **(e)** 3 **(f)** $3\sqrt{2}$

(f) none of the above

(ii) If this point was expressed in spherical coordinates, what would ρ be? **[2]**

(a) 1 **(b)** $\sqrt{3}$ **(c)** 2 (d) $2\sqrt{2}$ **(e)** 3 **(f)** $3\sqrt{2}$

(f) none of the above

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(iii) If this point was expressed in spherical coordinates, what would θ be? [3]

(a) 0 (b) $\frac{\pi}{6}$ **(c)** $\frac{\pi}{4}$ **(d)** $\frac{\pi}{3}$ **(e)** $\frac{\pi}{2}$ **(f)** π

(f) none of the above

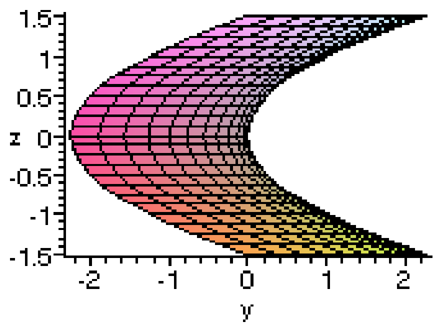
(iv) If this point was expressed in spherical coordinates, what would ϕ be? [3]

(a) $\frac{\pi}{6}$ **(b)** $\frac{\pi}{4}$ **(c)** $\frac{\pi}{3}$ **(d)** $\frac{\pi}{2}$ **(e)** $\frac{2\pi}{3}$ (f) $\frac{3\pi}{4}$

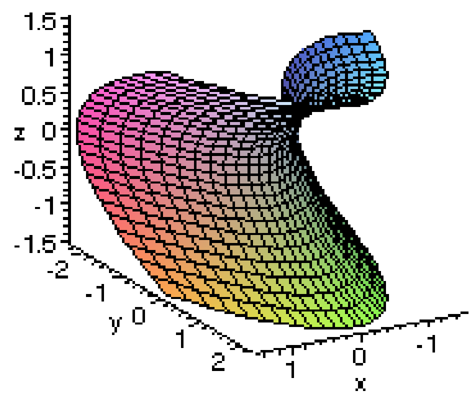
(g) $-\frac{\pi}{6}$ **(h)** $-\frac{\pi}{4}$ **(i)** $-\frac{\pi}{3}$ **(j)** $-\frac{\pi}{2}$ **(k)** $-\frac{2\pi}{3}$ **(l)** $-\frac{3\pi}{4}$

(m) none of the above

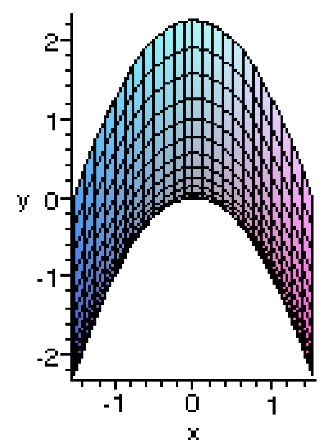
6. The surface shown to the right and below was plotted in Maple. It is an infinite surface, but it was cut off at $x = \pm 1.5$ and $z = \pm 1.5$. The plot is shown from various views, as seen in Maple as the 3-d plot is rotated.



View from x-axis



3-d View



View from z-axis

What is the equation of this surface?

(a) $x^2 + y^2 + z^2 = 1$ **(b)** $-x^2 + y^2 + z^2 = 1$

(c) $x^2 - y^2 + z^2 = 1$ **(d)** $x^2 + y^2 - z^2 = 1$

(e) $x^2 - y^2 - z^2 = 1$ **(f)** $-x^2 + y^2 - z^2 = 1$

(g) $-x^2 - y^2 + z^2 = 1$ **(h)** $-x^2 - y^2 - z^2 = 1$

(i) $x = y^2 + z^2$ **(j)** $y = x^2 + z^2$

(k) $x = y^2 - z^2$ **(l)** $y = x^2 - z^2$ **(m)** $x = -y^2 + z^2$

(n) $y = -x^2 + z^2$ **(o)** $z = x^2 + y^2$ **(p)** $z = x^2 - y^2$

(q) $z = -x^2 + y^2$ **(r)** none of the above

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Part B: Show All Your Work

Instructions: Work out the following problem, showing all your work. Part marks will be awarded even if the final answer is wrong.

7. Consider the space curve defined by the following:

$$x = \cos(t)$$
$$y = 2\sin(t)$$
$$z = \sqrt{3}\cos(t)$$

and assume there is a particle moving along this curve as a function of time t.

(a) Find the velocity vector and its magnitude. **[4]**

$$\vec{\mathbf{r}} = \left\langle \cos(t), 2\sin(t), \sqrt{3}\cos(t) \right\rangle$$

$$\vec{\mathbf{r}}' = \left\langle -\sin(t), 2\cos(t), -\sqrt{3}\sin(t) \right\rangle$$

$$\|\vec{\mathbf{r}}'\| = \sqrt{\sin^2(t) + 4\cos^2(t) + 3\sin^2(t)} = 2$$

(b) Find the acceleration vector and its magnitude. **[4]**

$$\vec{\mathbf{r}}'' = \left\langle -\cos(t), -2\sin(t), -\sqrt{3}\cos(t) \right\rangle = -\vec{\mathbf{r}}$$

$$\|\vec{\mathbf{r}}''\| = \sqrt{\cos^2(t) + 4\sin^2(t) + 3\cos^2(t)} = 2$$

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(c) What is the unit tangent vector $\vec{\mathbf{T}}$? **[2]**

$$\vec{\mathbf{T}} = \frac{\vec{\mathbf{r}}'}{\|\vec{\mathbf{r}}'\|} = \frac{1}{2}\vec{\mathbf{r}}' = \frac{1}{2}\left\langle -\sin(t), 2\cos(t), -\sqrt{3}\sin(t) \right\rangle$$

(d) What is the unit normal vector $\vec{\mathbf{N}}$? **[2]**

$$\vec{\mathbf{N}} = \frac{\vec{\mathbf{T}}'}{\|\vec{\mathbf{T}}'\|} = \frac{\frac{1}{2}\vec{\mathbf{r}}''}{\|\frac{1}{2}\vec{\mathbf{r}}''\|} = \frac{-\frac{1}{2}\vec{\mathbf{r}}}{\|\frac{1}{2}\vec{\mathbf{r}}\|} = -\frac{1}{2}\vec{\mathbf{r}} = -\frac{1}{2}\left\langle \cos(t), 2\sin(t), \sqrt{3}\cos(t) \right\rangle$$

(e) What is the binormal vector $\vec{\mathbf{B}}$? **[2]**

$$\vec{\mathbf{B}} = \vec{\mathbf{T}} \times \vec{\mathbf{N}} = -\frac{1}{4}\left\langle -\sin(t), 2\cos(t), -\sqrt{3}\sin(t) \right\rangle \times \left\langle \cos(t), 2\sin(t), \sqrt{3}\cos(t) \right\rangle$$

$$= -\frac{1}{4}\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -\sin(t) & 2\cos(t) & -\sqrt{3}\sin(t) \\ \cos(t) & 2\sin(t) & \sqrt{3}\cos(t) \end{vmatrix}$$

$$= -\frac{1}{4}\left\langle 2\sqrt{3}, 0, -2 \right\rangle = \left\langle -\frac{\sqrt{3}}{2}, 0, \frac{1}{2} \right\rangle$$

(f) What is the curvature of the curve? **[2]**

$$\kappa = \frac{\|\vec{\mathbf{T}}'\|}{\|\vec{\mathbf{r}}'\|} = \frac{1}{2}$$

(g) What is the arc length from t=0 to t=π? **[2]**

$$s = \int_0^t \|\vec{\mathbf{r}}'(t)\| dt = 2\int_0^t dt = 2\pi$$

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