Mathematics 251–3

Old Mid-Term Exam from Dr. Ryeburn

First Mid-Term Exam | Monday, September 30, 1996

1. In this question $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$ is a three-dimensional vector-valued function of a scalar variable, and a is a real number such that $\mathbf{r}(t)$ is defined in an open interval containing a (i.e., at a, and both to the left and to the right of a).

   (a) What does it mean to say that $\mathbf{r}(t)$ is continuous at $t = a$?

   (b) What do we mean by the derivative $\mathbf{r}'(a)$ of $\mathbf{r}(t)$ at $t = a$?

2. (a) Find either a vector equation or a triple of scalar (parametric) equations for the line L through the points $A = (2,5,1)$ and $B = (3,7,-1)$.

(b) How far is the point $(2,10,-3)$ from this line?

3. (a) Find an equation for the plane P through the points $A = (7, 0, 0)$, $B = (0, -7, -7)$, and $C = (6, -2, 0)$.

(b) How far is the point $(1, -9, 3)$ from this plane?

4. Consider the helix defined by $\mathbf{r}(t) = \langle 3\cos t, 3\sin t, 4t \rangle$.

(a) Find the unit tangent vector $\mathbf{T}(t)$.

(b) Find the unit normal vector $\mathbf{N}(t)$.

(c) Find the curvature $\kappa(t)$.

(d) Find the arc length for the quarter turn of the helix with $0 \le t \le \pi/2$.

5. (a) Prove that if $\mathbf{u}(t) = \langle u_1(t), u_2(t), u_3(t)\rangle$ and $\mathbf{v}(t) = \langle v_1(t), v_2(t), v_3(t)\rangle$ are differentiable vector-valued functions and $\varphi(t) = \mathbf{u}(t)\ \ \mathbf{v}(t)$ then $\varphi'(t) = \mathbf{u}'(t)\ \ \mathbf{v}(t) + \mathbf{u}(t)\ \ \mathbf{v}'(t)$.

(b) Use part (a) to show that if $\mathbf{u}(t) = \langle u_1(t), u_2(t), u_3(t)\rangle$ is a differentiable vector-valued function such that $|\mathbf{u}(t)| = 1$ for all t, then $\mathbf{u}(t)$ must be orthogonal to $\mathbf{u}'(t)$.