Is Two Better Than One?

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Most hold'em players know the odds against hitting an inside straight draw on a single card are roughly 11-to-1. This is the figure they have wired into their brains. Consequently, several people have expressed surprise at the 5.1-to-1 odds against making a straight by the river given a flop that allows an inside straight draw.

Last month we looked at the mathematics behind the big change in odds when considering drawing two cards successively versus one card at a time. So the mathematics has been explained but another question has arisen for the people mentioned above. The question revolves around the decision about whether to bet.

What they were telling me was that they would pay to see the turn if the pot was offering them better than 11-to-1 odds. Otherwise they would fold. However, after seeing the 5.1-to-1 figure, they were wondering whether it might not be profitable to draw for the straight in certain circumstances even if the pot is offering odds worse than 11-to-1. Let's take a look at their question.

We need to be very clear about what we are checking. If you might fold after the turn card, then you are allowing the possibility that you may pay to see only one card. In this case your decision must be based on the situation you face at the time with respect to drawing a single card. That is, if the pot is offering you less than 11-to-1 odds, then you should fold. (Of course, it is not quite that black and white as many players know. If you are facing several opponents who are likely to call even when you make your straight, then you don't need 11-to-1 odds to call. This involves the concept, introduced by David Sklansky, known as implied odds. We shall duck that issue for now.)

What we are going to examine is the strategy of seeing both the turn and the river cards no matter what happens on the turn. In other words, the player with the inside straight draw after the flop puts on blinders and says to himself, "I'm going all the way to hit this straight."

The exact probabilities are the following: 4/47 that he hits the straight on the turn, 86/1081 that he misses the straight on the turn but hits it on the river, and 903/1081 that he does not make the straight. We examine a typical situation to get a feel for what is going on.

Suppose our player P has three opponents, all of whom have bet x, and there is a total of $8x$ in the pot. Thus, P is getting 8-to-1 pot odds. If he hits the straight on the turn, we assume he wins no more chips. In other words, all players check and then fold when P bets. In reality, P may get callers if he bets because some of the opponents may have good draws themselves.

When P misses the straight on the turn card, we assume that each of his three opponents bets $2x$. Now there is $15x$ in the pot so that P is getting pot odds of 7.5-to-1. Note that the pot odds have decreased. If P catches a straight on the river, we again assume that he gets no callers when he bets. If P misses the straight, we assume he folds to any bet from one of his opponents.

It is now easy to verify that P's expected return following the strategy, subject to the assumptions, is $-683x/1081$. So P is giving up more than 60% of the big blind value by following such a strategy.

The preceding discussion involved a single example, but it contains the germ of a general proof that a player should not follow such a strategy. We present a rough outline of the proof. Suppose player P will get pot odds less than 11-to-1 even if all n opponents put in a single bet of x. In other words, when the action gets to P, he knows how much the players preceding him have put in, and if he assumes that all players acting after him also call a bet of x, the pot will have strictly less than $11x$ in it. This is not an unreasonable assumption because if any players acting after P fold his pot odds decrease. So if there was T in the pot before the flop and n opponents each bet x after the flop, then P is getting $(T + nx)$-to-x as his pot odds. We are assuming this is less than 11-to-1.

When P misses the straight on the turn, the best pot odds he will get for the next bet (assuming there are no raises) is $(T + (3n + 1)x)$-to-$2x$. A little algebra shows that this is at most $(T + nx)$-to-x because T is at least as big as $(n+1)x$. In other words, P's pot odds cannot increase when there is no raising.

Raising creates complications because players' decisions are sequential. For example, suppose there is $8x$ in the pot when it gets to player P. Player P now makes a mistake and calls the bet. A player betting after P raises to $2x$ and three players before P all call the raise. When the action gets back to P, there is now $17x$ in the pot and it costs P only an additional x to call. At this point he now is getting pot odds of 17-to-1 and should make the call. His earlier call was a mistake, but that bet is now in the pot and up for grabs like the other $16x$ in the pot. So it is correct to go for it now.

The latter illustrates that the situation is complicated in general. On the other hand, the main point is that when contemplating betting with respect to a straight draw, your decision should be based on what your pot odds are at that moment. One further comment is that all of the applies to limit hold'em because in no-limit hold'em opponents acting after you can manipulate pot odds dramatically and players need to be careful with drawing hands.