[3] **1.** Evaluate $\lim_{t\to 0} \dfrac{4-(t+2)^2}{t}$.

[4] **2.** Find a constant k such that $y = 2x - kx^2$ is a solution of the differential equation $xy' = y - x^2$.

[6] **3.** Let $f(x) = 1 - x^2$. Working directly from the definition of the derivative as a limit, verify the formula: $f'(a) = -2a$

[4] **4.** Let $f(x)$ denote the function defined by $f(x) = \begin{cases} \dfrac{1-e^x}{x} & \text{if } x \neq 0 \\ -1 & \text{if } x = 0. \end{cases}$

Show that $f(x)$ is continuous at $x = 0$.

[3] **5.** (a) Evaluate $\dfrac{d}{dx}\left(\dfrac{x^2-1}{x^2+1}\right)$ and simplify your answer.

[3] (b) Evaluate $\dfrac{d}{dx}\left[\tan\left(e^x+1\right)\right]$.

[5] (c) Given that $\sqrt{x^2+y^2} + \sqrt{xy} = 1$ find an expression for dy/dx in terms of x, y.
No simplification is necessary.

6. Consider the curve $\mathcal{C}$ described by the equation: $y^2 = x^3 + x^2$

[4] (a) Find the coordinates of the points of $\mathcal{C}$ at which the tangent lines are parallel to the x-axis.

[4] (b) Find the equations of the tangents to $\mathcal{C}$ at the origin $(0,0)$ and justify your answer.

[6] **7.** The following information is given about the function f:

$$f(x),\ f'(x),\ f''(x) \text{ are defined and continuous for all } x \neq 2$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = -1, \quad \lim_{x\to 2} f(x) = \infty$$

$$f(1) = f(7) = 0 \quad f'(-1/2) = 0 \quad f(0) = -7/4$$

$$1,\ 7 \text{ are the only zeros of } f(x);\ -1/2 \text{ is the only zero of } f'(x)$$

$$f''(x) < 0 \text{ for all } x \in (-\infty, -7/4),\ f''(-7/4) = 0$$

$$f''(x) > 0 \text{ for all } x \text{ in } (-7/4, 2) \text{ and all } x \text{ in } (2, \infty)$$

Using the axes provided on page 3 sketch the graph of $y = f(x)$ in a manner consistent with all the information.

No justification is required, but you may add comments if you wish.

[6] **8.** (a) The sun is directly overhead at noon and sets at 6 p.m. Assuming that θ, the angle of elevation of the sun above the horizon, changes at a constant rate, show that, at 4 p.m., the length of the shadow cast by a 6 metre high post is increasing at a rate of $\pi/30$ metres per minute.

[6] (b) The pressure P, volume V, and temperature T of the gas in a spherical balloon of radius r are related by the universal gas equation $PV = nRT$, where n is the number of moles of gas, and R is a constant. Here the temperature T is measured in Kelvins.

Let t be the elapsed time in hours. A variable x is said to be *increasing at a% per hour* if $\frac{1}{x}\frac{dx}{dt} = \frac{a}{100}$.

At the instant under consideration n is not changing, the temperature of the gas is increasing at 4% per hour, and the pressure of the gas is increasing at 1% per hour. Show that r, the radius of the balloon is also increasing at 1% per hour.

[4] **9.** (a) Find the linear approximation of the function $\sqrt[3]{x}$ at $x = 1000$ and use it to approximate $\sqrt[3]{1002}$.

[4] (b) Beginning with the initial estimate $x_0 = 10$ apply one step of Newton's Method to find an estimate for a root of $x^3 - 1002 = 0$.

10. Let $f(x) = \frac{1}{2}x^4 - x^3 - 6x^2 + 4x$.

$x = a$ is called a *critical point of* $f(x)$ if $f'(x) = 0$.

[2] (a) Find all the critical points of $f(x)$ given that one of them is $x = -2$.

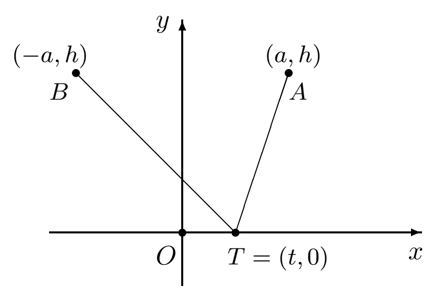
[3] (b) At which values of x, if any, does $f(x)$ have a local maximum?

At which values of x, if any, does $f(x)$ have a local minimum?

[3] (c) What is the largest interval on which $f(x)$ is concave down?

11. The point $T = (t, 0)$ varies on the x-axis. The points $A = (a, h)$ and $B = (-a, h)$ are fixed with a, $h > 0$. Define the function L by

$$L(t) = \text{length}(AT) + \text{length}(BT).$$



[4] (a) Express $L(t)$ in terms of t and the constants a, h.

[5] (b) Use calculus to show that $L(t)$ has an absolute minimum when $t = 0$.

[4] **12.** (a) Find the general antiderivative of $\sin 2x + \frac{2}{x^2}$.

[3] (b) Find a function y defined on $(0, \infty)$ such that $\frac{dy}{dx} = \frac{2x^2 + 1}{x}$, $y(1) = 0$.

13. A tank of brine has 1000 litre capacity and initially contains 50 kilograms of salt dissolved in water.

Brine is drawn from the tank at rate of 5 litres per minute and water is added to the tank at the same rate to maintain the volume of solution at 1000 litres.

The tank is well-stirred so that the concentration of salt is uniform at all times.

Let S denote the amount of salt (in kilograms) in the tank after t minutes.

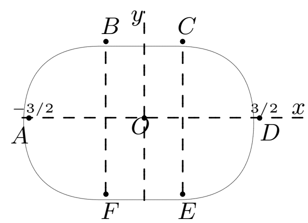
[2] (a) What is the approximate net change ΔS in the amount of salt in the tank in the time interval $[t, t + \Delta t]$ if Δt is small?

Write your answer as a constant multiple of $S\Delta t$.

[2] (b) Write down an equation relating dS/dt and S.

[4] (c) How many minutes pass before there are only 25 kilograms of salt in the tank?

[6] **14.** An oval plate is symmetric about its axes, which are shown as Ox, Oy in the figure. The midsection of the plate is a rectangle $BCEF$ of width 1 and height 2.



The arc AB of the bounding curve has the same shape as the arc $y = 2\sqrt{x} - x \quad (0 \leq x \leq 1)$. Indeed, the arc AB is obtained by translating the arc $y = 2\sqrt{x} - x \quad (0 \leq x \leq 1)$ horizontally $3/2$ units to the left.

Show the area of the plate is $16/3$.

Axes for Question 7