

GUIDE TO MATH 440 GALOIS THEORY

(UPDATED: SPRING 2013 IMIN CHEN)

DESCRIPTION

Galois Theory is, in short, an in depth study of algebraic field extensions. Its foundations were first discovered by a young French mathematician named Evariste Galois (1811-1832), who lived a short but rather colourful life during a time of political upheaval in France following the French Revolution. The theory was originally motivated by the problem of solving for the roots of polynomial equations using radical expressions, but the concepts and techniques of the theory have taken an importance of its own because it provides a powerful toolbox for understanding field extensions. One of the key features of the theory is the connection between a field extension and its group of symmetries, the so-called Galois correspondence, which appears in many other areas of mathematics such as topology and differential equations.

WHAT YOU NEED: PREREQUISITE MATH 340 OR 332

- Some basic algebraic notions from MATH 340 like fields, rings, groups, vector spaces over a field, extension of fields, a good sense of what these objects are from the concrete examples you saw in 340.
- Some maturity in constructing and recognizing valid mathematical proofs.
- Taking MATH 341 previously or concurrently is suggested – be prepared to work harder without MATH 341.

SOME THINGS YOU'LL LEARN:

- Important properties of field extensions, leading up to the notion of a Galois extension, and its Galois group.
- When it is possible or not possible to embed one field into another field.
- When it is possible or not possible to solve for the roots of a polynomial using radical expressions.
- Why it is not possible in general to solve for the roots of a polynomial of degree 5 or more using radical expressions.
- When it is possible or not possible to construct a regular n -gon using ruler and compass.
- A two page complete conceptual description of finite fields and their symmetries to complement and enhance the concrete descriptions learned in more application-oriented courses.
- Proof of and examples of the Galois correspondence.
- Examples of calculating the Galois groups of polynomials.
- Develop your algebra background in field theory (and to a lesser extent group and ring theory) to graduate level.

WHY LEARN GALOIS THEORY?

- Galois Theory is a prerequisite for further advanced graduate studies in Algebra and Number Theory.
- For example, the proof of Fermat's Last Theorem uses a kind of very advanced Galois Theory. Many of the major conjectures in Number Theory being investigated by research mathematicians today, such as Serre's conjecture, and Langlands conjecture, represent attempts at understanding more deeply the structure of Galois extensions of the rational numbers.
- For two of the Clay Millennium Prizes, the Birch-Swinnerton-Dyer Conjecture and the Riemann Hypothesis, the known cases which can be proved involve many advanced and sophisticated techniques. Galois Theory is basic to and assumed in these advanced techniques.
- Galois Theory is a powerful tool for studying field extensions, so understanding it provides a solid theoretical foundation for those involved in applications which use field extensions.
- Be one of the select few people in the world who know Galois Theory.