Solutions.

SFU-UBC-UNBC-UVic

CALCULUS CHALLENGE EXAMINATION

JUNE 07, 2012, 12:00 - 15:00

Name: _____ (please print)

family name *given name*

Signature: _____

Instructions:

1. Do not open this booklet until told to do so.
2. Write your name above in block letters and sign your exam.
3. Write your answer in the space provided below the question. If additional space is needed then use the back of the previous page. Your final answer should be simplified as far as is reasonable.
4. Make the method you are using clear in every case unless it is explicitly stated that no explanation is needed.
5. This exam has 15 questions on 15 pages (not including this cover page and the sheet of trigonometric identities). Once the exam begins please check to make sure your exam is complete.
6. **No** Books, papers, or electronic devices except a graphing calculator and the sheet of trigonometric identities attached to this examination shall be within the reach of a student during the examination.
7. **During the examination, communicating with, or deliberately exposing written papers to the view of, other examinees is forbidden.**

Question	Maximum	Score
1	8	
2	11	
3	8	
4	6	
5	8	
6	11	
7	6	
8	5	
9	6	
10	8	
11	6	
12	8	
13	8	
14	8	
15	8	
Total	115	

1. Compute the following limits.

[2] (a) $\lim_{x\to\infty} \dfrac{\ln x}{e^{2x}}$ $\left[\frac{\infty}{\infty}\right]$

$\overset{l'H}{=} \lim_{x\to\infty} \dfrac{\frac{1}{x}}{2e^{2x}} = \lim_{x\to\infty} \dfrac{1}{2x\,e^{2x}}$ $\boxed{= 0}$

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[3] (b) $\lim_{x\to 2}(x-1)^{\frac{1}{x-2}}$ $[1^{\infty}]$ $\quad L = \lim_{x\to 2} (x-1)^{1/x-2}$

$\ln L = \ln \lim_{x\to 2} (x-1)^{\frac{1}{x-2}} = \lim_{x\to 2} \ln(x-1)^{\frac{1}{x-2}}$

$\ln L = \lim_{x\to 2} \dfrac{\ln(x-1)}{x-2}$ $\left[\frac{0}{0}\right]$ $\overset{l'H}{=} \lim_{x\to 2} \dfrac{\frac{1}{x-1}}{1} = 1$

$\ln L = 1 \quad \therefore \quad L = e$

and $\lim_{x\to 2} (x-1)^{\frac{1}{x-2}}$ $\boxed{= e.}$

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[3] (c) $\lim_{x\to 6} \dfrac{\sqrt{x-2}-2}{\sqrt{x-6}}$ $\left[\frac{0}{0}\right]$

$= \lim_{x\to 6} \dfrac{(\sqrt{x-2}-2)(\sqrt{x-2}+2)}{\sqrt{x-6}(\sqrt{x-2}+2)} = \lim_{x\to 6} \dfrac{x-2-4}{(\sqrt{x-6})(\sqrt{x-2}+2)}$

$= \lim_{x\to 6} \dfrac{x-6}{(\sqrt{x-6})(\sqrt{x-2}+2)} = \lim_{x\to 6} \dfrac{(\sqrt{x-6})(\sqrt{x-6})}{(\sqrt{x-6})(\sqrt{x-2}+2)}$

$\boxed{= 0}$

2. Compute the indicated derivatives. You do not need to simplify your answer.

[3] (a) If $y = \dfrac{\cos x^2}{\sin x}$, find y'.

$$y' = \frac{(\sin x)(-\sin x^2)(2x) - \cos x^2 \cdot \cos x}{\sin^2 x}$$

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[4] (b) If $y = (1-3x)^{e^x}$, find $\dfrac{dy}{dx}$.

$$\ln y = \ln (1-3x)^{e^x} = e^x \ln(1-3x)$$

$$\frac{1}{y} y' = e^x \ln(1-3x) + e^x \cdot \frac{1}{1-3x}(-3)$$

$$y' = (1-3x)^{e^x} e^x \left[\ln(1-3x) - \frac{3}{1-3x} \right]$$

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[4] (c) Suppose that the functions f and g and their derivatives with respect to x have the values at $x = 0$ and $x = 1$ as shown in the table.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	5	2	1	3
1	7	-3	-2	-5

Evauluate $\dfrac{d(f(x+g(x)))}{dx}$ at $x = 0$.

$$= f'(x+g(x))\left[1 + g'(x)\right]$$

At $x=0$ $\quad f'(0+g(0))\left[1+g'(0)\right]$

$$= f'(0+1)\left[1+3\right]$$

$$= f'(1) \cdot 4$$

$$= (-3)(4) = \boxed{-12}$$

[2] **3.** (a) State the definition of the derivative of a function f AT A NUMBER a.

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad \text{if this limit exists}$$

OR

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad \text{if this limit exists}$$

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[4] (b) Let $f(x) = \dfrac{3x+1}{x-2}$. Use the DEFINITION of the derivative to find $f'(1)$.

$$f(1) = \frac{(3 \cdot 1) + 1}{1 - 2} = -4$$

$$f'(1) = \lim_{x \to 1} \frac{\frac{3x+1}{x-2} - (-4)}{x - 1} = \lim_{x \to 1} \frac{3x + 1 + 4(x-2)}{(x-2)(x-1)}$$

$$= \lim_{x \to 1} \frac{7x - 7}{(x-1)(x-2)} = \lim_{x \to 1} \frac{7(x-1)}{(x-1)(x-2)} = -7$$

$$\boxed{f'(1) = -7}$$

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[2] (c) Find the equation of the tangent line to $y = f(x)$ at $x = 1$.

When $x = 1$, $f(1) = -4$, the point is $(1, -4)$

Slope is $f'(1) = -7$

Equation is $y - (-4) = -7(x - 1)$

$$y + 4 = -7x + 7$$

$$\boxed{y = -7x + 3}$$

[2] **4.** (a) Find f if $f' = 2\cos x + 8x^3 - e^x$ and $f(0) = 7$

Taking the anti derivative.

$$f(x) = 2\sin x + 2x^4 - e^x + C$$

$$f(0) = 7 = 2\sin(0) + 2(0)^4 - e^0 + C$$

$$7 = -1 + C \quad \therefore \quad C = 8$$

$$f(x) = 2\sin x + 2x^4 - e^x + 8$$

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[4] (b) Find a curve $y = f(x)$ with the following properties:

1) $\frac{d^2y}{dx^2} = 6x$ and
2) Its graph passes through the point $(0, 1)$ and has a horizontal tangent line there.

$$y'' = 6x$$

$$y' = 3x^2 + C$$

$$y = x^3 + Cx + D$$

$(0,1)$ is on the curve

$$1 = 0^3 + (C)(0) + D \quad \therefore \quad D = 1$$

Horizontal tangent line at $x = 0$

$$y'(0) = 0 \Rightarrow 0 = 3(0)^2 + C \Rightarrow C = 0$$

$$\therefore \quad y = x^3 + 1$$