

Simon Fraser University

MATH 251

Summer 2004

Second Midterm Examination

Instructor: A. Belshaw

Date: July 7, 2004

Name: _____

Student number: _____

Signature: _____

Instructions

1. DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.
2. Fill in the information above.
3. This booklet contains 5 printed pages in addition to this cover page.
4. Do all your work in this test booklet. Show all your work. Use the backs of the pages if necessary.
5. No books, no notes, no calculators and no devices.
6. Students observed writing anything after the call to stop writing will be subject to summary penalties.

1	2	3	4	5	Total
10	8	14	12	6	50

- [4] 1. (a) Give the equation of the plane tangent to the surface $z = 4 - x^2 - y^2$ at the point $(1, 1, 2)$.

- [2] (b) What is the direction of most rapid increase of the function $f(x, y) = 4 - x^2 - y^2$ at the point $(3, 2)$?

- [4] (c) In what direction(s) is the directional derivative of $f(x, y) = 4 - x^2 - y^2$ at the point $(3, 2)$ equal to zero?

[8]

2. Use the method of Lagrange multipliers to find the closest point(s) on the surface $x^2yz = 1$ to the origin.

[8]

3. (a) Find and classify the critical points of

$$f(x, y) = 3x - 3y + x^2 - xy + 2y^2.$$

[6]

- (b) The area of an ellipse is given by $A = \pi ab$, where a and b are the lengths of the semiaxes and the equation of the ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Consider an cylinder of height $h = 3$, above an ellipse in the x-y plane with $a = 2$ and $b = 4$. How fast is the volume V of the cylinder increasing or decreasing if $\frac{dh}{dt} = 1$, $\frac{da}{dt} = 2$, and $\frac{db}{dt} = -\frac{1}{2}$?

[8]

4. (a) Use double integration to find the volume under the surface $z = x^2 + y$ over the region bounded by $x = y^2 - 2y$ and $y = x$.

[4]

- (b) Find

$$\int_0^\pi \int_0^\pi (e^x + \cos y) \, dx \, dy$$

[6]

5. Find the integral expression in polar coordinates for the volume of intersection of the cardioid cylinder $r = 1 + \cos \theta$ and the sphere $z^2 = 4 - x^2 - y^2$. Do not carry out the integration.