MATH 251-3, Spring 2006 **Simon Fraser University**

Midterm 2 7 March 2006, 5:30–6:20pm

Instructor: Ralf Wittenberg

Last Name:	
First Name:	
SFU ID:	
Signature:	

INSTRUCTIONS

1. PLEASE DO NOT OPEN THIS BOOKLET UNTIL INVITED TO DO SO.
2. Write your last name, first name(s) and student number in the box above in block letters, and sign your name in the space provided.
3. This exam contains 6 questions on 6 pages (after this title page). Once the exam begins please check to make sure your exam is complete.
4. The total time available is 50 minutes, and there are 50 points, so allow about a minute per point; for example, you should aim to spend about 10 minutes on a 10-point question. Attempt all problems!
5. This is a closed book exam. Only non-programmable scientific calculators are allowed.
6. Use the reverse side of the previous page if you need more room for your answer, and clearly indicate where the solution continues.
7. Show all your work, and explain your answers clearly.
8. Good luck!

Question	Maximum	Score
1	8	
2	8	
3	10	
4	8	
5	10	
6	6	
Total	50	

1. In each of the following cases find the limit, if it exists, or show that the limit does not exist:

   (a) [4 points]

$$\lim_{(x,y)\to(0,0)} \frac{3xy}{x^2+2y^2}$$

   (b) [4 points]

$$\lim_{(x,y)\to(0,0)} \frac{3xy^2}{x^2+2y^2}$$

2. (a) [6 points]
Classify the critical points of the function $x^3 + y^3 - 3xy$.

(b) [2 points]
Does the function have any global extrema? Justify your answer.

3. Consider the function

$$F(x, y, z) = x^2 z - \frac{y}{z^2} \qquad (z > 0).$$

(a) [5 points]
Find the directional derivative of F at the point $P(2, -3, 1)$ in the direction of the vector $\mathbf{v} = \mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$.

(b) [2 points]
Find the maximum rate of change of F at the point $P(2, -3, 1)$. In the direction of which unit vector $\mathbf{u}$ is the directional derivative a maximum?

(c) [3 points]
Find an equation to the tangent plane to the surface

$$F(x, y, z) = x^2 z - \frac{y}{z^2} = 7$$

at the point $P(2, -3, 1)$.

4. Let $w = \dfrac{x^2y^3}{z^4}$.

   (a) [4 points]
   Find the differential dw.

   (b) [4 points]
   Suppose x increases by 1%, y increases by 2% and z increases by 3%. Using your answer from (a): By approximately what percentage will the value of w increase or decrease?

5. Suppose that $w(x, y) = f(r)$, where $r = \sqrt{x^2 + y^2}$.

   (a) [6 points]
   Show that
   $$\frac{\partial^2 w}{\partial x^2} = \frac{y^2}{r^3}\frac{dw}{dr} + \frac{x^2}{r^2}\frac{d^2 w}{dr^2},$$
   where $dw/dr = df/dr = f'(r)$, $d^2w/dr^2 = d^2f/dr^2 = f''(r)$.

   (b) [4 points]
   Use the result from (a) to show that
   $$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{d^2 w}{dr^2} + \frac{1}{r}\frac{dw}{dr}.$$

6. [6 points]
   Use the method of Lagrange multipliers to find the maximum value of $f(x, y) = xy$ on the ellipse $4x^2 + 9y^2 = 36$.