

MATH 251
Simon Fraser University
Department of Mathematics
Burnaby Campus
Midterm Exam No. 2
Wednesday March 9, 2005
Instructor: Dr. Akeel Shah

DO NOT LIFT UP THE COVER PAGE UNTIL INSTRUCTED. Once the test begins, please check that all pages are intact. Answer all five questions, clearly showing your working. Each question is worth 8 marks. If you run out of space use the extra space on the back of each page, **CLEARLY INDICATING WHERE THE SOLUTION CONTINUES AND THE NUMBER OF THE QUESTION THAT YOU ARE ANSWERING**. Ordinary scientific calculators are allowed. The duration of the test is 40 minutes.

Name:

Question 1:

Question 2:

Question 3:

Question 4:

Question 5:

TOTAL:

1a) (4 marks) By making a transformation to polar coordinates, show that

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{\sqrt{x^2 + y^2}} = 0.$$

1b) (4 marks) By making the substitution $y = mx$, demonstrate that

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2 + xy}$$

does **not** exist.

2) (8 marks) By using the differential of a suitably defined function, approximate the number $(\sqrt{17} + \sqrt{99})^2$.

3) Let $w = \sqrt{x^2 + y^2 + z^2}$ and let $x = 3e^t \sin s$, $y = 3e^t \cos s$ and $z = 4e^t$.

(i) (2 marks) Name the independent and the intermediate variables.

(ii) (6 marks) Use the chain rule to find $\partial w / \partial t$ and $\partial w / \partial s$, expressing your answers as functions of t and s and simplifying as much as possible.

4) (8 marks) Find the maximum and minimum values of $f(x, y) = xy$ subject to the constraint $4x^2 + 9y^2 = 36$.

5) (8 marks) Find and classify the critical points of $f(x, y) = x^4 + y^4 - 4xy$.