MATH 251-3, Fall 2006 **Simon Fraser University**

Midterm 2 1 November 2006, 8:30–9:20am

Instructor: Ralf Wittenberg

Last Name:	______________________
First Name:	______________________
SFU ID:	______________________
Signature:	______________________

INSTRUCTIONS

1. PLEASE DO NOT OPEN THIS BOOKLET UNTIL INVITED TO DO SO.
2. Write your last name, first name(s) and SFU ID in the box above in block letters, and sign your name in the space provided.
3. This exam contains 5 questions on 5 pages (after this title page). Once the exam begins please check to make sure your exam is complete.
4. The total time available is 50 minutes, and there are 50 points, so allow about a minute per point; for example, you should aim to spend about 10 minutes on a 10-point question. Attempt all problems!
5. This is a closed book exam. Only non-programmable scientific calculators are allowed.
6. Use the reverse side of the previous page if you need more room for your answer, and clearly indicate where the solution continues.
7. Show all your work, and explain your answers clearly.
8. Good luck!

Question	Maximum	Score
1	14	
2	6	
3	7	
4	11	
5	12	
Total	50	

1. [14 points]
   The *acceleration* of a particle moving along a helical path is

$$\mathbf{a}(t) = \langle 4\cos 2t, 0, -4\sin 2t\rangle \ .$$

At time $t = 0$, the particle passes through the origin $(0, 0, 0)$ with initial velocity $\mathbf{v}(0) = \mathbf{r}'(0) = \langle 0, 3, 2\rangle$.

(a) Find the velocity $\mathbf{v}(t)$ and position $\mathbf{r}(t)$ of the particle, and also the speed of the particle, at time t.

(b) Reparametrize the curve with respect to arc length measured from the origin in the direction of increasing t.

(c) Find the tangential and normal components of the acceleration vector.

Question 1 continued on next page . . .

(d) For the path $\mathbf{r}(t)$ of the particle obtained in (a) above, find the curvature κ, and the unit tangent and principal normal vectors $\mathbf{T}$ and $\mathbf{N}$, at time t.

2. [6 points]
For which values of (x, y) is the following function continuous? Explain and justify your reasoning carefully.

$$f(x,y) = \begin{cases} \dfrac{xy^2\sqrt{1-x^2-y^2}}{x^2+2y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

3. [7 points]
The period $T(m,k)$ of a simple harmonic oscillator of mass m and spring constant k is given by

$$T = 2\pi\sqrt{\frac{m}{k}}\ .$$

(a) Find the differential of T.

(b) Estimate the percentage change in the period of oscillation if the mass m increases by 3% and the spring constant k decreases by 2%.

4. [11 points]
Let $z = f(x, y)$, where f has continuous second-order partial derivatives, and $x = r\cos\theta$, $y = r\sin\theta$.

(a) Find $\dfrac{\partial z}{\partial r}$ and $\dfrac{\partial z}{\partial \theta}$.

(b) Show that

$$\left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2 = \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 \overset{\text{(notation)}}{=} f_x^2 + f_y^2 \ .$$

(c) Find $\dfrac{\partial^2 z}{\partial r \partial \theta}$.

5. [12 points]
Let $F(x, y, z) = x^2 + y^2 + z^3 - 3xyz$, and let P be the point P$(1, 2, 1)$.

(a) Find the directional derivative of $F(x, y, z)$ at P$(1, 2, 1)$ in the direction of the vector $\mathbf{v} = \langle 2, 4, -3 \rangle$.

(b) In which direction does the value of F increase most rapidly at P, and what is the maximum rate of increase at P?

(c) Consider the level surface

$$F(x, y, z) = x^2 + y^2 + z^3 - 3xyz = 0 \ ,$$

which contains the point P$(1, 2, 1)$; on this surface, $z = z(x, y)$ is defined implicitly as a function of x and y near P.
Compute $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at P.

(d) Find the tangent plane to the level surface $F(x, y, z) = 0$ at P$(1, 2, 1)$.